VII.

On some Applications of the Method of Mechanical Quadratures.

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It is proposed, in this communication, to apply some of the known formulæ of the method of quadratures to several astronomical problems of interest, where, under certain conditions, their employment tends to accuracy and simplicity in computation, and offers methods entirely independent of those in common use.

I. Let A_0, $A_1 \ldots\ldots A_n$, be a series of values of any function of which t is the independent variable. For convenience of expression, we may call t the time for which A_0, $A_1 \ldots\ldots A_n$, are given. Supposing A to be computed numerically for the successive values of t, $t=0$, $t=\tau$, $t=2\tau \ldots\ldots t=n\tau$, τ being the equal interval between each succeeding value of t, and n the number of times for which A is found. We may then arrange t and A with its first, second, &c. differences, Δ^1, Δ^2, Δ^3, &c., in vertical columns, as follows: —

$$\begin{array}{llllll}
t=0 & A_0 & & & & \\
 & & \Delta_0^1 & & & \\
t=1\tau & A_1 & & \Delta_0^2 & & \\
 & & \Delta_1^1 & & \Delta_0^3 & \\
t=2\tau & A_2 & & \Delta_1^2 & & \Delta_0^4 \\
 & & \Delta_2^1 & & \Delta_1^3 & \\
t=3\tau & A_3 & & \Delta_2^2 & & \\
 & & \Delta_3^1 & & & \\
t=4\tau & A_4 & & & &
\end{array} \qquad (1)$$

It is plain that, as τ is decreased, $\frac{\Delta_0^1}{\tau}$, $\frac{\Delta_1^1}{\tau}$, &c., will approximate to the values of the first differential coefficients of A at the times $t=\frac{1}{2}\tau$, $t=\frac{3}{2}\tau$, &c., which they will accurately represent as long as the *third* differences of A, or Δ_0^3, Δ_1^3, &c., are insensible. If, then, we know A_0, and also the first differential coefficients of A for $t=\frac{1}{2}\tau$, $t=\frac{3}{2}\tau$, &c., these multiplied by τ, and substituted for Δ_0^1, $\Delta_1^1 \ldots\ldots \Delta_n^1$, in (1), will give, by

continued addition, A_1, $A_2 \ldots\ldots A_n$. And, in this manner, in the case supposed, that is, of Δ^3_0, $\Delta^3_1 \ldots\ldots \Delta^3_n$, being insensible, we shall obtain the total change of A between the limits $t=0$ and $t=n\tau$.

Before correcting this process, in order to take account of the third and higher orders of differences, it will be well to observe, that the effect of increasing n, or the number of intervals into which the whole time is divided, is to diminish Δ^2, Δ^3, &c., relatively to Δ^1, nearly in the proportion of the corresponding powers of n. Thus, when the number of intervals is doubled, Δ^1 has about one half its previous value, Δ^2 one quarter, Δ^3 one eighth, and so on. The intervals, therefore, can always be diminished so as to make Δ^3 insensible compared with Δ^1. But it is better, in most cases, to allow Δ^3 to be a small quantity, and to correct A for its influence, and for that of higher orders of differences.

Arranging the numerical values of the first differential coefficients * of A, represented by V, with their differences, D^1, D^2, &c., in vertical columns, as has already been done with A, we have, taking for convenience τ for the unit of time, or $\tau=1$:—

$$(2)\qquad \begin{array}{lllllll}
t=-\frac{3}{2} & V_{-\frac{3}{2}} & & & & \\
 & & D^1_{-\frac{3}{2}} & & & \\
=-\frac{1}{2} & V_{-\frac{1}{2}} & & D^2_{-\frac{3}{2}} & & \\
 & & D^1_{-\frac{1}{2}} & & D^3_{-\frac{3}{2}} & \\
=+\frac{1}{2} & V_{+\frac{1}{2}} & & D^2_{-\frac{1}{2}} & & D^4_{-\frac{3}{2}} \\
 & & D^1_{+\frac{1}{2}} & & D^3_{-\frac{1}{2}} & \\
=+\frac{3}{2} & V_{+\frac{3}{2}} & & D^2_{+\frac{1}{2}} & & \\
 & & D^1_{+\frac{3}{2}} & & & \\
=+\frac{5}{2} & V_{+\frac{5}{2}} & & & &
\end{array}$$

If we put a, b, c, &c., for the first, second, &c. differential coefficients of A_0, we shall have at any time $t=n\tau=n$,

$$(3)\qquad A_n = A_0 + a\,t + \tfrac{b}{1.2}\,t^2 +, \text{\&c.}$$

$$(4)\qquad V_n = a + b\,t + \tfrac{c}{1.2}\,t^2 +, \text{\&c.}$$

By making successively $t=\frac{1}{2}$, $t=\frac{3}{2}$, &c., in the expression for V_n, we obtain by elimination a, b, c, &c., in terms of V and numerical coefficients, and these substituted in the value of A_n give

$$(5)\qquad \Delta^1_n = A_{n+1} - A_n = V_{n+\frac{1}{2}} + \tfrac{1}{24}\,D^2_{n-\frac{1}{2}} - \tfrac{17}{5760}\,D^4_{n-\frac{3}{2}} + \tfrac{367}{967680}\,D^6_{n-\frac{5}{2}} -, \text{\&c.}$$

And thence

$$(6)\qquad A_n - A_0 = \text{sum of all the quantities } V_{\frac{1}{2}} \ldots\ldots V_{n-\frac{1}{2}} + \tfrac{1}{24}\left(D^1_{n-\frac{1}{2}} - D^1_{-\frac{1}{2}}\right) - \tfrac{17}{5760}\left(D^3_{n-\frac{3}{2}} - D^3_{-\frac{3}{2}}\right) + \tfrac{367}{967680}\left(D^5_{n-\frac{5}{2}} - D_{-\frac{5}{2}}\right) -, \text{\&c.}$$

* The usual notation for first and second differential coefficients is not employed in this paper, from its resemblance to that here used for finite differences.

It will be noticed, on reference to (2), that the differences $D^2_{n-\frac{1}{2}}$, $D^4_{n-\frac{3}{2}}$, &c., are on the same horizontal line with $V_{n+\frac{1}{2}}$. The intervals may usually be taken small enough to enable the computer to neglect differences of V of a higher order than the second. Then the total variation of A between the limits for which V is known is the sum of all the values of $V+\frac{1}{24}(D^1_{n+\frac{1}{2}}-D^1_{-\frac{1}{2}})$.

Where the intermediate values of A are required, $\varDelta^1_0$, $\varDelta^1_1$, $\varDelta^1_2$ ..... $\varDelta^1_{n-1}$, are first found, and these, by continual addition, give A_1, A_2 ..... A_n; and in this case (6) serves to test the correctness of A_n-A_0, which should be the same by both processes.

If, instead of the *first*, we have the *second* differential coefficients of A, denoted by F_{-2}, F_{-1}, F_0 ..... F_n, for $t=-2$, $t=-1$, $t=0$ ..... $t=n$, these may be arranged as before, with their differences d^1, d^2, &c.:—

$$\begin{array}{rllllll}
t=-2 & F_{-2} & & & & & \\
 & & d^1_{-2} & & & & \\
=-1 & F_{-1} & & d^2_{-2} & & & \\
 & & d^1_{-1} & & d^3_{-2} & & \\
=\ \ 0 & F_0 & & d^2_{-1} & & d^4_{-2} & \\
 & & d^1_0 & & d^3_{-1} & & \\
=\ \ 1 & F_1 & & d^2_0 & & & \\
 & & d^1_1 & & & & \\
=\ \ 2 & F_2 & & & & &
\end{array} \tag{7}$$

By a process similar to that used in finding $\varDelta^1_n$, we obtain

$$\varDelta^2_n=F_{n+1}+\tfrac{1}{12}\,d^2_n-\tfrac{1}{240}\,d^4_{n-1}+\tfrac{31}{60480}\,d^6_{n-2}-\text{, \&c.} \tag{8}$$

$$\varDelta^1_n-\varDelta^1_0=\text{sum of all the quantities } F_1\,.....\,F_n+\tfrac{1}{12}(d^1_n-d^1_0)-\tfrac{1}{240}(d^3_{n-1}-d^3_{-1})+\tfrac{31}{60480}(d^5_{n-2}-d^5_{-2})-\text{, \&c.}; \tag{9}$$

and $A_n-A_0=$ sum of all the quantities $\varDelta^1_0$ ..... $\varDelta^1_{n-1}$.

When, therefore, A_0 and $\varDelta^1_0$ are known, with the values of F for each interval, we obtain by (8) $\varDelta^2_0$, $\varDelta^2_1$ ..... $\varDelta^2_{n-2}$, thence by successive additions $\varDelta^1_0$, $\varDelta^1_1$ ..... $\varDelta^1_{n-1}$, and finally A_0, A_1 ..... A_n.

From the values of F_0, F_1 ..... F_{n-1}, the first differential coefficients of A may also be found by using (5), and considering F as the first differential coefficient of V; $V_{-\frac{1}{2}}$ being first known, by (5) we find $V_{+\frac{1}{2}}$, $V_{+\frac{3}{2}}$ ..... $V_{+n-\frac{1}{2}}$.

It will be observed, that when $V_{\frac{1}{2}}$ ..... $V_{n-\frac{1}{2}}$ are given, and it is required to find A_n for any time, we must first know A_0, and then we have $A_n=(A_n-A_0)+A_0$, (A_n-A_0) being derived from (6). But when F_1 ..... F_{n-1} are known, and it is required to find A_n, we must know $\varDelta^1_0$ as well as A_0.

The following expressions derived from (3) and (4) give $\varDelta^1_0$ in terms of F and its differences, when V_0 or $V_{\frac{1}{2}}$ are known:—

$$\varDelta^1_0=V_0+\tfrac{1}{2}F_0+\tfrac{1}{6}\,d^1_0-\tfrac{1}{24}\,d^2_0+\tfrac{1}{45}\,d^3_0-\tfrac{7}{480}\,d^4_0+\tfrac{107}{10080}\,d^5_0-\tfrac{199}{24192}\,d^6_0+\text{, \&c.}; \tag{10}$$

$$\varDelta^1_0=V_{\frac{1}{2}}+\tfrac{1}{24}\,d^1_0-\tfrac{7}{5760}\,d^3_{-1}+\tfrac{43}{322560}\,d^5_{-2}-\text{, \&c.}; \tag{11}$$

which will be afterwards referred to.

(5) and (8) are two of the elementary formulæ in the method of mechanical quadratures. Usually the convergence of the differences of V and F, and the smallness of their coefficients, allow of the differences above the second or third order being neglected.

It is an objection which applies with some force to the practical use of this method, that a numerical error committed in any part of the work is, by the nature of the process, disseminated over a great mass of computations. The best safeguard for accuracy is the use of suitable tests and checks at proper intervals. The chief expenditure of labor will generally be in computing the series of numerical values of V and F; any considerable accidental errors in these quantities will show themselves in taking the differences. As these quantities are independently calculated, there is no tendency here to an aggregation of errors. But in the summation of V or F, an error of addition in forming any one value of A affects also those succeeding it. A security against this will be to use (6) and (9) after A_n has been independently found by the successive addition of $\Delta_0^1 \ldots\ldots \Delta_{n-1}^1$.

It is useful to know what degree of accuracy in V or F will be sufficient, in a given case, to compute A_n accurately to a certain number of decimal places. If V be known to the same number of decimal places (not significant figures) with A, and in every instance has an error e in its last decimal place, and always with the same sign, then the possible error of $A_n = n\,e$. And a similar error e' in F gives the possible error of $A_n = \frac{n\,(n-1)}{1.2}\,e'$.

But it is plain that these possible errors are highly improbable. Regarding it as equally probable that the last figure of each value of V or F is too large or too small by one unit, or exactly right, when the number of intervals or n is given, the comparative probabilities of the errors of A_n, or of $\Delta_n^1 - \Delta_0^1$, being within certain limits, will be nearly expressed by

$$\frac{1^2 \,.\, 2^2 \,.\, 3^2 \ldots\ldots h^2}{1 \,.\, 2 \,.\, 3 \ldots\ldots a \,.\, 1 \,.\, 2 \,.\, 3 \ldots\ldots b};$$

where a and b are respectively the number of times that the last decimal place of V or of F must be too large and too small by unity, in order to produce the given errors in A_n or $\Delta_n^1 - \Delta_0^1$; the whole number of instances in which V or F will be too large or too small being assumed as about two thirds of n, or $a + b = \frac{2}{3}\,n$, and $h = \frac{1}{3}\,n = \frac{a+b}{2}$, a, b, and h being the nearest whole numbers to their exact values, and unity the probability that $a = b$, or that the error of $A_n = 0$, which is its most probable value.

When $A_n - A_0$, or $\Delta_n^1 - \Delta_0^1$, increases pretty uniformly, V will be of the same order with $\frac{1}{n}\,(A_n - A_0)$ and F of the same order with $\frac{1}{n}\,(\Delta_n^1 - \Delta_0^1)$, and consequently the number of significant figures in V or F will usually be much less than in A. But these

conditions are not susceptible of any precise statement, being affected by a variety of circumstances.

We now proceed to the application of the method of quadratures to some examples in astronomy.

II. *Computation of the Radius Vector r in Orbits of any Eccentricity.*

The differential expression which seems best fitted for this purpose is

$$F r^2 = \frac{2\mu}{r} - \frac{2\mu}{a}. \tag{12}$$

$F r^2$ being the second differential coefficient of r^2, and $\frac{1}{a}$ the reciprocal of the semi-axis of the orbit, which is $=0$ in the parabola, and negative in the hyperbola; μ is the sun's mass + the mass of the planet, which, when τ is the number of mean solar days in each interval, and the mass of the body is neglected, may be expressed by $\mu = k^2 \tau^2$, log. $k = 8.2355814$.

We shall suppose that r_0^2 and $\Delta^1 r_0^2 = r_1^2 - r_0^2$ are given, with the semi-axis of the orbit, and that it is required to compute a series of values of r from $t=0$ to $t=n\tau$; τ being taken such that $\frac{2\mu}{r}$ is very small.

The first step is to compute F_0 and F_1 from (12) using a, r_0, and r_1. As $\frac{2\mu}{a}$ is a constant, each value of F requires only the calculation of $\frac{2\mu}{r}$ to be repeated for each interval. Neglecting, at first, $\frac{1}{12} d_0^2$ in (8), which is of the order of the fourth differences of r^2, F_1 added to Δ_0^1 gives Δ_1^1, and thence r_2^2, very nearly; this value of r_2^2 used in (12) gives a very exact value of F_2, because whatever error there may be in r_2, it is multiplied in F_2, by the coefficient μ, which we may make as small as we please by diminishing τ.

Arranging now F_0, F_1, and F_2 in a vertical column, and taking the differences,

$$\begin{array}{llllllll}
t=0 & F_0 & & & r_0^2 & & \\
 & & d_0^1 & & & \Delta_0^1 & \\
t=1 & F_1 & & d_0^2 & r_1^2 & & \Delta_0^2 \\
 & & d_1^1 & & & \Delta_1^1 & \\
t=2 & F_2 & & & r_2^2 & &
\end{array}$$

we obtain d_0^2, with which we correct the first value of r_2^2. This correction, where the intervals between r_0^2, r_1^2, &c., do not exceed two or three days, or as many degrees of heliocentric motion, will be exceedingly small, and wholly insensible on F_2, which may next be used to find $r_3^2 = r_2^2 + \Delta_1^1 + F_2$; it being seldom necessary to notice the differences of F above the second.

After F is known for two or three intervals, r^2 follows for the remaining intervals without intervening approximation. In this manner r^2 may be computed with accuracy in the seventh place of decimals, with logarithmic tables of five places, each value re-

quiring but one new logarithm. Table I., at the end of this article, contains numerical values of $\frac{2\mu}{r}$, with r^2 for the argument, supposing the interval to be one day.

It is evident that this method cannot be advantageously employed, unless the intermediate values of r between r_0 and r_n are wanted.

In the following example, r is the radius vector of Halley's Comet for each Greenwich mean midnight, from August 1st to August 20th, 1835; using the elements given in the *Nautical Almanac* for 1839. The influence of the second and higher differences of F is insensible, so that $\varDelta^2_n = F_{n+1}$, and the whole series is calculated directly from Table I., without using logarithms after computing the constants r_0^2, $\varDelta^1 r_0^2$, and $\frac{2\mu}{a}$.

$$r_0^2 = 4.019789\ 0; \qquad \varDelta^1 r_0^2 = -0.055698\ 0; \qquad -\frac{2\mu}{a} = -0.000032\ 7.$$

$\frac{2\mu}{r}$ is taken from Table I., with the argument r_n^2, the value of r^2, next to r_n^2, being $r^2_{n+1} = r_n^2 + \varDelta^1_{n-1} + F_n$. The correctness of the series may be tested by computing r_n^2 for August 20.5 by the usual methods.

	r^2	$\varDelta^1 r^2$	$\varDelta^2 r^2$	$-\frac{2\mu}{a}$	$\frac{2\mu}{r}$
1835, August 1.5	4.019789 0	—55698 0			
2.5	3.964091 0	55433 5	+264 5=	—32 7	+297 2
3.5	3.908657 5	55166 9	266 6=	"	+299·3
4.5	3.853490 6	54898 1	268 8=	"	+301 5
5.5	3.798592 5	54627 3	270 8=	"	+303 5
6.5	3.743965 2	54354 1	273 2=	"	+305 9
7.5	3.689611 1	54078 8	275 3=	"	+308 0
8.5	3.635532 3	53801 1	277 7=	"	+310 4
9.5	3.581731 2	53521 1	280 0=	"	+312 7
10.5	3.528210 1	53238 9	282 2=	"	+314 9
11.5	3.474971 2	52954 1	284 8=	"	+317 5
12.5	3.422017 1	52667 0	287 1=	"	+319 8
13.5	3.369350 1	52377 3	289 7=	"	+322 4
14.5	3.316972 8	52085 0	292 3=	"	+325 0
15.5	3.264887 8	51790 2	294 8=	"	+327 5
16.5	3.213097 6	51492 7	297 5=	"	+330 2
17.5	3.161604 9	51192 6	300 1=	"	+332 8
18.5	3.110412 3	50889 8	302 8=	"	+335 5
19.5	3.059522 5	—50584 2	+305 6=	"	+338 3
20.5	3.008938 3				

(13) The substitution of $V r^2 = \frac{d.r^2}{dt} = \frac{2e\sqrt{\mu}}{\sqrt{p}}\, r \sin. v = 2e\sqrt{\mu a}\, \sin. u$

in equations (10) or (11) affords an accurate value of $\varDelta^1 r_0^2$ when $F r_0^2$ and its differences are known. Or it may be obtained from the expressions,

(14)
$$r_1 - r_0 = \frac{2e}{p} r_1 r_0 \sin. \tfrac{1}{2}(v_1 + v_0) \sin. \tfrac{1}{2}(v_1 - v_0) = 2ae \sin. \tfrac{1}{2}(u_1 + u_0) \sin. \tfrac{1}{2}(u_1 - u_0),$$

$$\varDelta^1 r_0^2 = (r_1 + r_0)(r_1 - r_0),$$

e and p being the eccentricity and semi-parameter of the orbit, and u and v the eccentric and true anomalies.

III. *Computation of Heliocentric Coördinates x, y, and z, by Quadratures.*

The method of proceeding so nearly resembles that for finding r, that a few words will serve to introduce it.

According as x, y, or z is to be found, we have

$$Fx = -\frac{\mu}{r^3}x, \quad Fy = -\frac{\mu}{r^3}y, \quad Fz = -\frac{\mu}{r^3}z. \tag{15}$$

With the constants x_0, y_0, z_0, $\varDelta^1 x_0$, $\varDelta^1 y_0$, and $\varDelta^1 z_0$, and the series for r^2 as already computed, we obtain F_0 and F_1, and thence x_2, y_2, and z_2; Fx_2, Fy_2, and Fz_2 give x_3, y_3, and z_3, and these again x_4, y_4, and z_4, and so on; noticing the higher orders of differences when necessary.

A very useful test of the accuracy of the whole series for r, x, y, and z is afforded by putting

$$r_n^2 = x_n^2 + y_n^2 + z_n^2. \tag{16}$$

For if this condition is satisfied, it indicates that every preceding value of r, x, y, and z, from $t = 0$ to $t = n\tau$, is correct.

In calculating x, y, and z to seven places of decimals, with logarithms of five figures, the number of days in each interval should not exceed $2r$. For the maximum value of F is $= -\frac{\mu}{r^2} = -k^2\left(\frac{\tau}{r}\right)^2$. Since $k^2 = 0.0003$ about, if $\frac{\tau}{r} > 2$, F will not be given with accuracy beyond seven places of decimals with five-figure logarithms.

In the following example, x, y, and z are the coördinates of Halley's comet, referred to the equator, for Greenwich mean midnight, from August 1.5 to August 9.5, 1835. The necessary constants, determined from the elements in the *Nautical Almanac* for 1839, are as follows: —

$$\begin{array}{lll} \text{August 1.5,} & x_0 = +\,0.9934592 & \varDelta_0^1 x = +\,0.0007486\ 5 \\ & y_0 = +\,1.6196067 & \varDelta_0^1 y = -\,0.0163206\ 0 \\ & z_0 = +\,0.6400825 & \varDelta_0^1 z = -\,0.0035933\ 7 \end{array}$$

The corrections for second differences of F are insensible.

1835,	x	$\varDelta^1 x$	$-\frac{\mu}{r^3}x$	y	$\varDelta^1 y$	$-\frac{\mu}{r^3}y$	z	$\varDelta^1 z$	$-\frac{\mu}{r^3}z$
Aug. 1.5	+0.9934592 0			+1.6196067 0			+0.6400825 0		
		+7486 5			—163206 0			—35933 7	
2.5	.9942078 5		—372 9	.6032861 0		—601 2	.6364891 3		—238 7
		7113 6			163807 2			36172 4	
3.5	.9949192 1		381 2	.5869053 8		607 8	.6328718 9		242 4
		6732 4			164415 0			36414 8	
4.5	.9955924 5		389 6	.5704638 8		614 5	.6292304 1		246 2
		6342 8			165029 5			36661 0	
5.5	.9962267 3		398 2	.5539609 3		621 1	.6255643 1		250 1
		5944 6			165650 6			36911 1	
6.5	.9968211 9		407 2	.5373958 7		628 1	.6218732 0		254 1
		5537 4			166278 7			37165 2	
7.5	.9973749 3		416 5	.5207680 0		635 1	.6181566 8		258 1
		5120 9			166913 8			37423 3	
8.5	.9978870 2		—426 1	.5040766 2		—642 2	.6144143 5		—262 3
		+4694 8			—167556 0			—37685 6	
9.5	+0.9983565 0			+1.4873210 2			+0.6106457 9		

When x, y, and z are wanted for a long period, it will be well to guard against an accumulation of errors in the series, by computing a new set of constants for another epoch, directly from the elements of the orbit.

As it is of importance that $\Delta^1 x_0$, $\Delta^1 y_0$, and $\Delta^1 z_0$ should be known beyond the last decimal place of x_0, y_0, and z_0, the following equations, or others of a similar nature, are useful in determining them.

The projections of the radius vector in an ellipse upon the principal axes are

$$r \sin. v = \sqrt{ap} \sin. u, \text{ and } r \cos. v = a (\cos. u - e).$$

And since these axes have a fixed direction, the coördinates of a body moving in an elliptical orbit, referred to any plane passing through the sun, are

$$(17) \quad x = A_0 + A_1 \sin. u + A_2 \sin.^2 \tfrac{1}{2} u, \quad y = B_0 + B_1 \sin. u + B_2 \sin.^2 \tfrac{1}{2} u, \quad z = C_0 + C_1 \sin. u + C_2 \sin.^2 \tfrac{1}{2} u;$$

whence

$$(18) \quad \begin{aligned} \Delta^1 x_0 &= x_1 - x_0 = 2 A_1 \cos. \tfrac{1}{2}(u_1 + u_0) \sin. \tfrac{1}{2}(u_1 - u_0) + A_2 \sin. \tfrac{1}{2}(u_1 + u_0) \sin. \tfrac{1}{2}(u_1 - u_0); \\ \Delta^1 y_0 &= y_1 - y_0 = 2 B_1 \cos. \tfrac{1}{2}(u_1 + u_0) \sin. \tfrac{1}{2}(u_1 - u_0) + B_2 \sin. \tfrac{1}{2}(u_1 + u_0) \sin. \tfrac{1}{2}(u_1 - u_0); \\ \Delta^1 z_0 &= z_1 - z_0 = 2 C_1 \cos. \tfrac{1}{2}(u_1 + u_0) \sin. \tfrac{1}{2}(u_1 - u_0) + C_2 \sin. \tfrac{1}{2}(u_1 + u_0) \sin. \tfrac{1}{2}(u_1 - u_0); \end{aligned}$$

A_0, A_1, &c., being constant quantities, determined by giving particular values to u. Putting for u its values when $v = 0°$, and $v = 90°$, we find

$$(19) \quad \begin{aligned} A_0 &= X, & A_1 &= \frac{X'}{\sqrt{1-e^2}}, & A_2 &= -\frac{2X}{1-e}; \\ B_0 &= Y, & B_1 &= \frac{Y'}{\sqrt{1-e^2}}, & B_2 &= -\frac{2Y}{1-e}; \\ C_0 &= Z, & C_1 &= \frac{Z'}{\sqrt{1-e^2}}, & C_2 &= -\frac{2Z}{1-e}. \end{aligned}$$

X, Y, and Z are the coördinates of the perihelion point of the orbit; X', Y', and Z', those of the point of intersection of the parameter with the orbit, that is, a point 90° from the perihelion.

In the parabola the above expressions become

$$(20) \quad x = X + X' \tan. \tfrac{1}{2} v - X \tan.^2 \tfrac{1}{2} v, \quad y = Y + Y' \tan. \tfrac{1}{2} v - Y \tan.^2 \tfrac{1}{2} v, \quad z = Z + Z' \tan. \tfrac{1}{2} v - Z \tan.^2 \tfrac{1}{2} v.$$

$$(21) \quad \begin{aligned} \Delta^1 x_0 &= x_1 - x_0 = X' \frac{\sin. \tfrac{1}{2}(v_1 - v_0)}{\cos. \tfrac{1}{2} v_1 \cos. \tfrac{1}{2} v_0} - X \frac{\sin. \tfrac{1}{2}(v_1 - v_0) \sin. \tfrac{1}{2}(v_1 + v_0)}{\cos.^2 \tfrac{1}{2} v_1 \cos.^2 \tfrac{1}{2} v_0}; \\ \Delta^1 y_0 &= y_1 - y_0 = Y' \frac{\sin. \tfrac{1}{2}(v_1 - v_0)}{\cos. \tfrac{1}{2} v_1 \cos. \tfrac{1}{2} v_0} - Y \frac{\sin. \tfrac{1}{2}(v_1 - v_0) \sin. \tfrac{1}{2}(v_1 + v_0)}{\cos.^2 \tfrac{1}{2} v_1 \cos.^2 \tfrac{1}{2} v_0}; \\ \Delta^1 z_0 &= z_1 - z_0 = Z' \frac{\sin. \tfrac{1}{2}(v_1 - v_0)}{\cos. \tfrac{1}{2} v_1 \cos. \tfrac{1}{2} v_0} - Z \frac{\sin. \tfrac{1}{2}(v_1 - v_0) \sin. \tfrac{1}{2}(v_1 + v_0)}{\cos.^2 \tfrac{1}{2} v_1 \cos.^2 \tfrac{1}{2} v_0}. \end{aligned}$$

By differentiating the above equations, and substituting for $\frac{du}{dt}$ and $\frac{dv}{dt}$ the proper values, we find the following expressions for the differential coefficients of x, y, and z:

$$(22) \quad Vx = (A_1 \cos. u + \tfrac{1}{2} A_2 \sin. u) \frac{k}{r\sqrt{a}}, \quad Vy = (B_1 \cos. u + \tfrac{1}{2} B_2 \sin. u) \frac{k}{r\sqrt{a}}, \quad Vz = (C_1 \cos. u + \tfrac{1}{2} C_2 \sin. u) \frac{k}{r\sqrt{a}}.$$

And in the parabola,

$$(23) \quad Vx = (X' - 2 X \tan. \tfrac{1}{2} v) \frac{k}{r\sqrt{p}}, \quad Vy = (Y' - 2 Y \tan. \tfrac{1}{2} v) \frac{k}{r\sqrt{p}}, \quad Vz = (Z' - 2 Z \tan. \tfrac{1}{2} v) \frac{k}{r\sqrt{p}}.$$

IV. *To find by Quadratures the Partial Differentials of the Geocentric Right Ascension and Declination of a Comet or Planet relatively to the Elements of its Orbit.*

This is a problem of frequent occurrence where it is required to find the orbit which best satisfies a large number of observations. Its nature is such that no solution can be expected, for the attainment of which a very considerable amount of labor is not necessary.

After the approximate elements of the orbit are known, instead of varying these elements themselves, as in the usual method of finding equations of condition for their correction, it will be an equivalent process to use x_0, y_0, z_0, Vx_0, Vy_0, and Vz_0 for the assumed constants of the orbit, and to form equations of condition for determining their corrections. The best epoch for x_0, y_0, &c., will usually be at nearly midway between the extreme observations. The coefficients of the unknown quantities in these equations are the partial differentials of α and θ, the geocentric right ascension and declination, relatively to x_0, y_0, z_0, Vx_0, Vy_0, and Vz_0, and they must be known before the equations can be solved.

Having with the above constants found x, y, and z from $t=0$ to $t=n\tau$, by the process of Sect. III., each constant is to be separately increased by the addition of a quantity δx_0, δy_0, δz_0, δVx_0, δVy_0, and δVz_0, small enough to allow of its square being neglected. Commencing with δx_0, from the equations

$$r^2 = x^2 + y^2 + z^2, \quad \tfrac{1}{2} V r^2 = xVx + yVy + zVz, \tag{24}$$

$$\tfrac{1}{2} F r^2 = \tfrac{\mu}{r} - \tfrac{\mu}{a} = (Vx)^2 + (Vy)^2 + (Vz)^2 - \tfrac{\mu}{r}, \quad Fx = -\tfrac{\mu}{r^3} x, \quad Fy = -\tfrac{\mu}{r^3} y, \quad Fz = -\tfrac{\mu}{r^3} z, \tag{25}$$

we have

$$\delta r_0^2 = 2\, x_0\, \delta x_0, \quad V \delta r_0^2 = 2\, Vx_0\, \delta x_0, \quad F\delta r_0^2 = -2\, Fx_0\, \delta x_0 = -\delta\left(\tfrac{\mu}{a}\right), \tag{26}$$

in which Vx_0 and Fx_0 correspond to the intervals between δr_0^2 and δr_1^2 which may be considerably larger than when the total value of x is computed. As Vx, Vy, and Vz will vary from their previous values after the first interval, we have, excepting when $t=0$, $F\delta r^2 = -\frac{\mu}{r^3}\delta r^2 - \delta\left(\frac{2\mu}{a}\right)$; and since $2F\delta r_0^2 = -4Fx_0\,\delta x_0 = -\delta\left(\frac{2\mu}{a}\right)$ is a con- (27) stant for all values of r, and $\frac{\mu}{r^3}$ is known, $F\delta r^2$ will be known when δr^2 is known. And the process of finding δr^2 from $t=0$ to $t=n\tau$ is very much the same as that employed in Sect. II. Neglecting at first in (10) δd_0^1 and higher orders of the differences of $F\delta r^2$, which will be very small, we obtain δr_1^2 nearly, which affords an accurate value of $F\delta r_1^2 = -\frac{\mu}{r_1^3}\delta r_1^2 - \delta\frac{2\mu}{a}$. Again, neglecting δd_0^2, we find Fdr_2^2, and so on, correcting for the differences of $F\delta r^2$ as far as may be necessary to give sufficiently accurate values of $\delta r_0^2 \ldots\ldots \delta r_n^2$. For the corresponding equations for the changes of y and z, they are to be substituted for x in (26).

When $V x_0$ is varied, we have from (24),

(28) $$\delta r_0^2 = 0, \qquad V \delta r_0^2 = 2 x_0 \delta V x_0, \qquad F \delta r_0^2 = 4 V x_0 \delta V x_0 = -\delta \left(\tfrac{2\mu}{a}\right),$$

(29) and for any other epoch excepting $t = 0$, $F \delta r^2 = -\frac{\mu}{r^3} \delta r^2 - \delta \left(\frac{2\mu}{a}\right)$; and since for all values of t, $4 V x_0 \; \delta V x_0 = -\delta \left(\frac{2\mu}{a}\right)$ is constant as before, and $\frac{\mu}{r^3}$ is already known, δr^2 will follow from equations (27) and (28), when $V x_0$ is varied, precisely as from equations (26) and (27) when x_0 is varied.

Having thus found δr^2 from $t = 0$ to $t = n \tau$, it remains to determine δx, δy, and δz for the same period.

From (25) we find

(30) $$F \delta x = F x \left(\frac{\delta x}{x} - \frac{3}{2} \frac{\delta r^2}{r^2}\right), \qquad F \delta y = F y \left(\frac{\delta y}{y} - \frac{3}{2} \frac{\delta r^2}{r^2}\right), \qquad F \delta z = F z \left(\frac{\delta z}{z} - \frac{3}{2} \frac{\delta r^2}{r^2}\right),$$

in which all the quantities are known excepting δx, δy, and δz. When x_0 is varied, we have

(31) $$\delta x_0 = \delta x_0, \qquad V \delta x_0 = 0, \qquad F \delta x_0 = F x_0 \left(\frac{\delta x_0}{x_0} - \frac{3}{2} \frac{\delta r_0^2}{r_0^2}\right).$$

We may, therefore, find approximate values of δx_1 and δx_2, which give accurate values of $F \delta x_1$ and $F \delta x_2$, which may be used to find δx_3, and so on, correcting for the differences of $F \delta x$ as far as is necessary, and we thus obtain δx from $t = 0$ to $t = n \tau$.

Again, when x_0 is varied,

(32) $$\begin{array}{lll} \delta y_0 = 0, & V \delta y_0 = 0, & F \delta y_0 = -F y_0 \frac{3}{2} \frac{\delta r_0^2}{r_0^2}; \\ \delta z_0 = 0, & V \delta z_0 = 0, & F \delta z_0 = -F z_0 \frac{3}{2} \frac{\delta r_0^2}{r_0^2}; \end{array}$$

from which we proceed to find δy_1, δz_1, &c., which will be small compared with δx for several intervals.

When $V x_0$ is varied, we have, for $t = 0$,

(33) $$\begin{array}{lll} \delta x_0 = 0, & V \delta x_0 = \delta V x_0, & F \delta x_0 = 0; \\ \delta y_0 = 0, & V \delta y_0 = 0, & F \delta y_0 = 0; \\ \delta z_0 = 0, & V \delta z_0 = 0, & F \delta z_0 = 0; \end{array}$$

and at any other epoch the equations (30), by which δx, δy, and δz may be found from $t = 0$ to $t = n \tau$.

When y_0 and $V y_0$, z_0 and $V z_0$, are varied, the operations are precisely analogous to those employed for δx_0 and $\delta V x_0$.

Finally, the correctness of each series is tested by putting at $t = n \tau$

(34) $$\tfrac{1}{2} \delta r^2 = x \delta x + y \delta y + z \delta z.$$

The changes of α and θ, the geocentric right ascension and declination, are, $\mathbf{x}$ and $\mathbf{z}$ being the geocentric coördinates,

(35) $$\begin{aligned} \sin. \delta \alpha &= \frac{\cos. \alpha}{\mathbf{x}} (\delta y \cos. \alpha - \delta x \sin. \alpha); \\ \sin. \delta \theta &= \frac{\sin.^2 \theta}{\mathbf{z}} (\delta z \cot. \theta - \delta y \sin. \alpha - \delta x \cos. \alpha). \end{aligned}$$

The equations of condition are then to be formed and solved as usual.

V. *Calculation of Perturbations by Quadratures.*

The usual application of quadratures to the calculation of perturbations is in the summation of the momentary variations of the elements of the disturbed orbit, which, it has been found, may be expressed with much simplicity, considering the general intricacy of the problem, in terms of the coördinates x, y, and z, and their first differential coefficients, combined with the disturbing forces. For this purpose the positions of all the disturbing bodies of the system are computed for the middle of each interval, and also, either directly, or indirectly by the use of auxiliary angles, the coördinates of the disturbed body x, y, and z, with their first differential coefficients for the same epochs. From these, combined with the disturbing forces from $t = \frac{1}{2}$ to $t = n - \frac{1}{2}$, are derived the momentary variations of each element for the same times, and thence, by using (5) to sum up their values, will result the alteration of each of the six elements at the end of each interval.

After the perturbations of the elements for each interval have been ascertained, the changes which they introduce into x, y, and z are investigated, leading finally to the perturbations in geocentric right ascension and declination. Where the change of the elements is large, the process is sometimes repeated with corrected values of x, y, and z, in order to include the squares of the disturbing forces.

If we commence with the constants x_0, y_0, z_0, Vx_0, Vy_0, and Vz_0 for $t = 0$, we have

$$Fx = -\frac{\mu}{r^3}x - \mu A, \qquad Fy = -\frac{\mu}{r^3}y - \mu B, \qquad Fz = -\frac{\mu}{r^3} - \mu C; \tag{36}$$

in which

$$\begin{aligned} \mu A &= m'\left(\frac{x - x'}{\varrho'^3} + \frac{x'}{r'^3}\right) + m''\left(\frac{x - x''}{\varrho''^3} + \frac{x''}{r''^3}\right) + \text{\&c.}, \\ \mu B &= m'\left(\frac{y - y'}{\varrho'^3} + \frac{y'}{r'^3}\right) + m''\left(\frac{y - y''}{\varrho''^3} + \frac{y''}{r''^3}\right) + \text{\&c.}, \\ \mu C &= m'\left(\frac{z - z'}{\varrho'^3} + \frac{z'}{r'^3}\right) + m''\left(\frac{z - z''}{\varrho''^3} + \frac{z''}{r''^3}\right) + \text{\&c.}, \end{aligned} \tag{37}$$

are denominated the disturbing forces, μ being the sum of the masses of the sun and the disturbed body; m', m'', &c., the masses of the disturbing bodies; x', x'', &c., their coördinates from the sun; r', ϱ', r'', ϱ'', &c., their distances from the sun and from the disturbed body, of which the heliocentric coördinates and distance are x, y, z, and r. m', m'', &c., it will be noticed, being referred to the same unit with μ, are always very small compared with it, and therefore the second terms in (36) are much less than the first.

Making always $r^2 = x^2 + y^2 + z^2$, instead of finding it by Sect. II., because the pro- (38)
cess there given cannot here be adopted, the semi-axis being no longer constant, we may compute Fx_0, Fx_1, &c., and thence find $\Delta^1 x_0$, $\Delta^1 y_0$, $\Delta^1 z_0$, x_1, x_2, &c., as in Sect. III., with this difference, that Fx, Fy, and Fz contain the terms μA, μB, and μC, and that r^2 is to be found from (38).

Regarding Fx, Fy, and Fz as the first differential coefficients of Vx, Vy, and Vz, we obtain from the series $Fx_0 \ldots\ldots Fx_n$, $Fy_0 \ldots\ldots Fy_n$, $Fz_0 \ldots\ldots Fz_n$, by using (5), Vx, Vy, and Vz for $t=(n-\frac{1}{2})\tau$. As we know x, y, and z for the same epoch, we can compute the elements of the orbit at $t=(n-\frac{1}{2})\tau$, and the differences between them and those at $t=0$ are their perturbations from $t=0$ to $t=(n-\frac{1}{2})\tau$. When $n\tau$ is very large, which will depend on the heliocentric motion of the disturbed body, it will not be safe to use logarithms of five figures to find Fx, Fy, and Fz, and the labor of computing them is thereby much increased.

In order to avoid this difficulty, if we denote by δ_n the whole perturbation of the quantity to which it is affixed up to the moment $t=n\tau$, so that δx_n is the difference between the actual value of x_n and that which it would have had if from the time $t=0$ the sun's force alone had been exerted upon it, we may suppose the whole period to be divided into portions such that from $t=0$ to $t'=0$, from $t'=0$ to $t''=0$, &c., the squares of δx, δy, and δz, and their products with m', m'', &c., are insensible; we then have from (36)

$$(39)\quad F\delta x=Fx\left(\frac{\delta x}{x}-\frac{3}{2}\frac{\delta r^2}{r^2}\right)-\mu A,\qquad F\delta y=Fy\left(\frac{\delta y}{y}-\frac{3}{2}\frac{\delta r^2}{r^2}\right)-\mu B,\qquad F\delta z=Fz\left(\frac{\delta z}{z}-\frac{3}{2}\frac{\delta r^2}{r^2}\right)-\mu C,$$

by which δx, δy, and δz may be determined from $t=0$ to $t'=0$, independently of the exact values of x, y, and z. For although the unknown quantities δx, δy, and δz enter the expressions (39), the coefficient $\frac{\mu}{r^3}$ may be made as small as we please, it being of the order of the square of the heliocentric motion of the disturbed body in one interval. So that, by taking τ sufficiently small, we make the first terms of $F\delta x$, $F\delta y$, and $F\delta z$ very small compared with μA, μB, and μC, and this will be the case for several of the first intervals.

$$\text{At } t=0,\quad (40)\quad \begin{array}{lll} \delta x_0=0, & V\delta x_0=0, & F\delta x_0=-\mu A_0, \\ \delta y_0=0, & V\delta y_0=0, & F\delta y_0=-\mu B_0, \\ \delta z_0=0, & V\delta z_0=0, & F\delta z_0=-\mu C_0, \end{array}$$

(10) gives $\Delta^1\delta x_0=-\frac{1}{2}\mu A_0$, neglecting d_0^1, and thence

$$\delta x_1=-\tfrac{1}{2}\mu A_0,\qquad \delta y_1=-\tfrac{1}{2}\mu B_0,\qquad \delta z_1=-\tfrac{1}{2}\mu C_0.$$

With these we obtain $F\delta x_1$, $F\delta y_1$, and $F\delta z_1$, noticing that $\frac{1}{2}\delta r^2=x\delta x+y\delta y+z\delta z$. After correcting δx_1, δy_1, and δz_1 as far as necessary for the differences of $F\delta x$, $F\delta y$, and $F\delta z$, the computation of the rest of the series will be direct, and conducted on the same principles as have already been detailed in the preceding sections. From δx, δy, and δz, we pass to the perturbations in geocentric right ascension and declination by means of (35).

$F\delta x$, $F\delta y$, and $F\delta z$ being the first differential coefficients of δVx, δVy, and δVz,

the summation of $F\delta x_0 \ldots\ldots F\delta x_n$, $F\delta y_0 \ldots\ldots F\delta y_n$, $F\delta z_0 \ldots\ldots F\delta z_n$ by (5) will give δVx, δVy, and δVz at the epoch $t=(n-\frac{1}{2})\tau$; for this time we also know δx, δy, and δz, and these give, added to Vx, Vy, Vz, x, y, and z computed from the elements at $t=0$, the actual values of Vx, Vy, Vz, x, y, and z for the epoch $t=(n-\frac{1}{2})\tau$ or $t'=0$; we may then proceed to find δx, δy, and δz from $t'=0$ to $t''=0$ precisely as between $t=0$ and $t'=0$.

There seems to be no ready mode of testing δx, δy, and δz without computing δr^2 independently; we might then put $\frac{1}{2}\delta r^2 = x\delta x + y\delta y + z\delta z$. But the expression for (41) $F\delta r^2$ under disturbing forces is rather complicated. It may be well to notice, however, that in an undisturbed orbit the quantity $q = xVy - yVx$ is constant. Under the influence of disturbing forces it is shown in most treatises on perturbations that the momentary variation of q is

$$Vq = y\mu A - x\mu B \tag{42}$$

at $t=0$, $\delta q=0$; then using (5) we may sum up the known values of Vq from $t=\frac{1}{2}\tau$ to $t=(n-\frac{1}{2})\tau$, whence we find δq at $t=(n-\frac{1}{2})\tau$, and we ought to have

$$q + \delta q = xVy - yVx, \tag{43}$$

x, y, Vx, and Vy being the actual values of x, y, &c., found as above at $t=(n-\frac{1}{2})\tau$.

Assuming for the elements of the moon's orbit at Greenwich, midnight, June 15th, 1846,

Longitude of Descending Node,	36° 04′ 47″.0	True Eq., June 15, 1846.
Perigee to the Descending Node,	107° 35′ 07″.6	
Inclination,	5° 08′ 35″.4	
Mean Longitude,	57° 45′ 40″.7	
Mean Daily Motion,	13° 24′ 30″.66	
Log. Eccentricity,	8.6487211	
Mean Distance = $\frac{1}{400}$ of the sun's,		

δx, δy, and δz may be found for a few days by the method explained above. The exact values of $F\delta x$, $F\delta y$, and $F\delta z$ are

$$F\delta x = M\left(\frac{x-\delta x}{(r-\delta r)^3} - \frac{x}{r^3}\right) - MA;\ F\delta y = M\left(\frac{y-\delta y}{(r-\delta r)^3} - \frac{y}{r^3}\right) - MB;\ F\delta z = M\left(\frac{z-\delta z}{(r-\delta r)^3} - \frac{z}{r^3}\right) - MC;$$

$$(2r-\delta r)\delta r = (2x-\delta x)\delta x + (2y-\delta y)\delta y + (2z-\delta z)\delta z;$$

M being the sum of the masses of the earth and moon, and MA, MB, and MC the sun's disturbing forces, those of the planets being neglected. Taking the ecliptic for the plane of x and y at $t=0=$ June 15.5, we have

$$\delta x_0 = 0, \quad \delta y_0 = 0, \quad \delta z_0 = 0; \quad V\delta x_0 = 0, \quad V\delta y_0 = 0, \quad V\delta z_0 = 0;$$
$$F\delta x_0 = -MA, \quad F\delta y_0 = -MB, \quad F\delta z_0 = -MC;$$

and thence δx, δy, and δz in parts of the moon's mean distance for intervals of twelve hours, as follows: —

1846,	$F\delta x$	δx	$F\delta y$	δy	$F\delta z$	δz
June 15.5	—.0000684 9	—.0000000 0	—.0000021 5	+.0000000 0	—.0000043 5	—.0000000 0
" 16.0	685 2	00341 9	+.0000127 5	14 1	38 7	20 9
" 16.5	697 8	01370 0	267 5	154 9	33 9	80 5
" 17.0	715 6	03096 4	386 3	561 4	28 1	173 9
" 17.5	725 8	05537 8	477 0	1351 8	20 6	295 2
" 18.0	713 4	08703 1	539 6	2616 9	—.0000011 1	436 9
" 18.5	663 5	12578 7	582 2	2419 9	+.0000000 3	589 5
" 19.0	566 8	17113 9	619 0	6804 6	13 1	741 6
" 19.5	—.0000418 5	—.0022211 7	+.0000670 6	+.0009808 4	+.0000025 6	—.0000880 5

Which correspond to the following values of the perturbations in longitude and latitude.

	$\delta \alpha$	$\delta \vartheta$
1846, June 15.5	+0′ 00″.00	— 0″.00
16.0	0 00.01	0.04
16.5	0 05.43	0.35
17.0	0 24.52	1.36
17.5	1 03.11	3.37
18.0	2 05.63	7.17
18.5	3 34.07	10.95
19.0	5 27.69	15.80
19.5	+7 42.71	—20.70

For longer intervals, the terms producing the principal inequalities might be excluded from $F\delta x$, $F\delta y$, and $F\delta z$.

Table I. contains numerical values of $\frac{2\mu}{r}$ with the argument r^2. The interval adopted for the value of μ is one mean solar day. The use of the table is explained on pp. 193, 194.

Table II. contains logarithmic values of $\frac{\mu}{r^3}$, with the argument r^2, and is useful in calculating heliocentric coördinates.

TABLE I.

r^2	$\frac{2\mu}{r}$	Dif.	r^2	$\frac{2\mu}{r}$	Dif.	r^2	$\frac{2\mu}{r}$	Dif.	r^2	$\frac{2\mu}{r}$	Dif.	r^2	$\frac{2\mu}{r}$	Dif.	r^2	$\frac{2\mu}{r}$	Dif.
	0.000			0.000			0.000			0.000			0.000			0.000	
0.351	9989 4		0.424	9088 8		0.497	8394 9		0.570	7839 0		0.643	7380 5		0.716	6994 1	
.352	9975 2	14 2	.425	9078 1	10 7	.498	8386 5	8 4	.571	7832 1	6 9	.644	7374 8	5 7	.717	6989 2	4 9
.353	9961 1	14 1	.426	9067 5	10 6	.499	8378 1	8 4	.572	7825 3	6 8	.645	7369 1	5 7	.718	6984 3	4 9
.354	9947 0	14 1	.427	9056 9	10 6	.500	8369 7	8 4	.573	7818 5	6 8	.646	7363 4	5 7	.719	6979 4	4 9
.355	9933 0	14 0	.428	9046 3	10 6	.501	8361 3	8 4	.574	7811 6	6 9	.647	7357 7	5 7	.720	6974 6	4 8
.356	9919 0	14 0	.429	9035 7	10 6	.502	8353 0	8 3	.575	7804 8	6 8	.648	7352 0	5 7	.721	6969 8	4 8
.357	9905 1	13 9	.430	9025 2	10 5	.503	8344 7	8 3	.576	7798 1	6 7	.649	7346 3	5 7	.722	6964 9	4 9
.358	9891 2	13 9	.431	9014 7	10 5	.504	8336 5	8 2	.577	7791 3	6 8	.650	7340 7	5 6	.723	6960 1	4 8
.359	9877 4	13 8	.432	9004 3	10 4	.505	8328 2	8 3	.578	7784 6	6 7	.651	7335 1	5 6	.724	6955 3	4 8
.360	9863 7	13 7	.433	8994 1	10 2	.506	8320 0	8 2	.579	7777 8	6 8	.652	7329 4	5 7	.725	6950 5	4 8
.361	9850 0	13 7	.434	8983 7	10 4	.507	8311 8	8 2	.580	7771 1	6 7	.653	7323 8	5 6	.726	6945 7	4 8
.362	9836 4	13 6	.435	8973 4	10 3	.508	8303 6	8 2	.581	7764 5	6 6	.654	7318 2	5 6	.727	6940 9	4 8
.363	9822 8	13 6	.436	8963 1	10 3	.509	8295 5	8 1	.582	7757 8	6 7	.655	7312 6	5 6	.728	6936 2	4 7
.364	9809 3	13 5	.437	8952 9	10 2	.510	8287 3	8 2	.583	7751 2	6 6	.656	7307 1	5 5	.729	6931 4	4 8
.365	9795 9	13 4	.438	8942 6	10 3	.511	8279 2	8 1	.584	7744 5	6 7	.657	7301 5	5 6	.730	6926 7	4 7
.366	9782 5	13 4	.439	8932 3	10 3	.512	8271 1	8 1	.585	7737 9	6 6	.658	7295 9	5 6	.731	6921 9	4 8
.367	9769 1	13 4	.440	8922 1	10 2	.513	8263 1	8 0	.586	7731 3	6 6	.659	7290 4	5 5	.732	6917 2	4 7
.368	9755 8	13 3	.441	8912 0	10 1	.514	8255 1	8 0	.587	7724 7	6 6	.660	7284 9	5 5	.733	6912 5	4 7
.369	9742 6	13 2	.442	8901 9	10 1	.515	8247 1	8 0	.588	7718 1	6 6	.661	7279 4	5 5	.734	6907 8	4 7
.370	9729 5	13 1	.443	8891 8	10 1	.516	8239 1	8 0	.589	7711 6	6 5	.662	7273 9	5 5	.735	6903 1	4 7
.371	9716 3	13 2	.444	8881 8	10 0	.517	8231 0	8 1	.590	7705 0	6 6	.663	7268 4	5 5	.736	6898 4	4 7
.372	9703 3	13 0	.445	8871 8	10 0	.518	8223 0	8 0	.591	7698 5	6 5	.664	7262 9	5 5	.737	6893 7	4 7
.373	9690 2	13 1	.446	8861 8	10 0	.519	8215 1	7 9	.592	7692 0	6 5	.665	7257 4	5 5	.738	6889 1	4 6
.374	9677 3	12 9	.447	8851 8	10 0	.520	8207 2	7 9	.593	7685 5	6 5	.666	7252 0	5 4	.739	6884 4	4 7
.375	9664 4	12 9	.448	8841 9	9 9	.521	8199 3	7 9	.594	7679 0	6 5	.667	7246 6	5 4	.740	6879 8	4 6
.376	9651 5	12 9	.449	8832 1	9 8	.522	8191 5	7 8	.595	7672 6	6 4	.668	7241 2	5 4	.741	6875 2	4 6
.377	9638 7	12 8	.450	8822 3	9 8	.523	8183 6	7 9	.596	7666 1	6 5	.669	7235 8	5 4	.742	6870 6	4 6
.378	9625 9	12 8	.451	8812 5	9 8	.524	8175 8	7 8	.597	7659 7	6 4	.670	7230 4	5 4	.743	6865 9	4 7
.379	9613 [illegible]	12 7	.452	8802 7	9 8	.525	8168 0	7 8	.598	7653 3	6 4	.671	7225 0	5 4	.744	6861 3	4 6
.380	9600 6	12 6	.453	8793 0	9 7	.526	8160 2	7 8	.599	7646 9	6 4	.672	7219 6	5 4	.745	6856 7	4 6
.381	9587 9	12 7	.454	8783 3	9 7	.527	8152 5	7 7	.600	7640 5	6 4	.673	7214 3	5 3	.746	6852 2	4 5
.382	9575 4	12 5	.455	8773 6	9 7	.528	8144 8	7 7	.601	7634 1	6 4	.674	7208 9	5 4	.747	6847 6	4 6
.383	9562 9	12 5	.456	8764 0	9 6	.529	8137 0	7 8	.602	7627 8	6 3	.675	7203 6	5 3	.748	6843 0	4 6
.384	9550 4	12 5	.457	8754 4	9 6	.530	8129 3	7 7	.603	7621 5	6 3	.676	7198 2	5 4	.749	6838 5	4 5
.385	9538 0	12 4	.458	8744 8	9 6	.531	8121 6	7 7	.604	7615 2	6 3	.677	7192 9	5 3	.750	6833 9	4 6
.386	9525 6	12 4	.459	8735 3	9 5	.532	8114 0	7 6	.605	7608 9	6 3	.678	7187 6	5 3	.751	6829 4	4 5
.387	9513 3	12 3	.460	8725 8	9 5	.533	8106 4	7 6	.606	7602 6	6 3	.679	7182 3	5 3	.752	6824 8	4 6
.388	9501 1	12 2	.461	8716 3	9 5	.534	8098 8	7 6	.607	7596 3	6 3	.680	7177 0	5 3	.753	6820 3	4 5
.389	9488 8	12 3	.462	8706 9	9 4	.535	8091 2	7 6	.608	7590 1	6 2	.681	7171 7	5 3	.754	6815 8	4 5
.390	9476 7	12 1	.463	8697 5	9 4	.536	8083 6	7 6	.609	7583 8	6 3	.682	7166 5	5 2	.755	6811 3	4 5
.391	9464 5	12 2	.464	8688 1	9 4	.537	8076 0	7 6	.610	7577 6	6 2	.683	7161 2	5 3	.756	6806 8	4 5
.392	9452 4	12 1	.465	8678 8	9 3	.538	8068 5	7 5	.611	7571 4	6 2	684	7156 0	5 2	.757	6802 3	4 5
.393	9440 4	12 0	.466	8669 5	9 3	.539	8061 0	7 5	.612	7565 2	6 2	.685	7150 7	5 3	.758	6797 8	4 5
.394	9428 4	12 0	.467	8660 2	9 3	.540	8053 6	7 4	.613	7559 0	6 2	.686	7145 5	5 2	.759	6793 3	4 5
.395	9416 5	11 9	.468	8650 9	9 3	.541	8046 2	7 4	.614	7552 8	6 2	.687	7140 3	5 2	.760	6788 8	4 5
.396	9404 6	11 9	.469	8641 7	9 2	.542	8038 7	7 5	.615	7546 7	6 1	.688	7135 1	5 2	.761	6784 3	4 5
.397	9392 8	11 8	.470	8632 5	9 2	.543	8031 3	7 4	.616	7540 5	6 2	.689	7129 9	5 2	.762	6779 9	4 4
.398	9380 9	11 9	.471	8623 3	9 2	.544	8024 0	7 3	.617	7534 4	6 1	.690	7124 8	5 1	.763	6775 4	4 5
.399	9369 2	11 7	.472	8614 2	9 1	.545	8016 6	7 4	.618	7528 3	6 1	.691	7119 6	5 2	.764	6771 0	4 4
.400	9357 5	11 7	.473	8605 1	9 1	.546	8009 3	7 3	.619	7522 2	6 1	.692	7114 5	5 1	.765	6766 6	4 4
.401	9345 8	11 7	.474	8596 0	9 1	.547	8001 9	7 4	.620	7516 1	6 1	.693	7109 4	5 1	.766	6762 1	4 5
.402	9334 2	11 6	.475	8587 0	9 0	.548	7994 6	7 3	.621	7510 0	6 1	.694	7104 2	5 2	.767	6757 7	4 4
.403	9322 6	11 6	.476	8577 9	9 1	.549	7987 4	7 2	.622	7504 0	6 0	.695	7099 1	5 1	.768	6753 3	4 4
.404	9311 1	11 5	.477	8568 9	9 0	.550	7980 1	7 3	.623	7497 9	6 1	.696	7094 0	5 1	.769	6748 9	4 4
.405	9299 6	11 5	.478	8559 9	9 0	.551	7972 9	7 2	.624	7491 9	6 0	.697	7088 9	5 1	.770	6744 5	4 4
.406	9288 1	11 5	.479	8551 0	8 9	.552	7965 6	7 3	.625	7485 9	6 0	.698	7083 8	5 1	.771	6740 1	4 4
.407	9276 7	11 4	.480	8542 1	8 9	.553	7958 4	7 2	.626	7479 9	6 0	.699	7078 7	5 1	.772	6735 7	4 4
.408	9265 3	11 4	.481	8533 2	8 9	.554	7951 3	7 1	.627	7474 0	5 9	.700	7073 7	5 0	.773	6731 4	4 3
.409	9254 0	11 3	.482	8524 4	8 8	.555	7944 1	7 2	.628	7468 0	6 0	.701	7068 6	5 1	.774	6727 0	4 4
.410	9242 7	11 3	.483	8515 5	8 9	.556	7937 0	7 1	.629	7462 1	5 9	.702	7063 6	5 0	.775	6722 7	4 3
.411	9231 4	11 3	.484	8506 8	8 7	.557	7929 8	7 2	.630	7456 2	5 9	.703	7058 5	5 1	.776	6718 4	4 3
.412	9220 2	11 2	.485	8498 0	8 8	.558	7922 7	7 1	.631	7450 3	5 9	.704	7053 5	5 0	.777	6714 1	4 3
.413	9209 1	11 1	.486	8489 3	8 7	.559	7915 7	7 0	.632	7444 4	5 9	.705	7048 5	5 0	.778	6709 7	4 4
.414	9197 9	11 2	.487	8480 6	8 7	.560	7908 6	7 1	.633	7438 5	5 9	.706	7043 5	5 0	.779	6705 4	4 3
.415	9186 8	11 1	.488	8471 9	8 7	.561	7901 6	7 0	.634	7432 6	5 9	.707	7038 5	5 0	.780	6701 1	4 3
.416	9175 8	11 0	.489	8463 2	8 7	.562	7894 5	7 1	.635	7426 8	5 8	.708	7033 5	5 0	.781	6696 8	4 3
.417	9164 8	11 0	.490	8454 6	8 6	.563	7887 5	7 0	.636	7421 0	5 8	.709	7028 6	4 9	.782	6692 6	4 2
.418	9153 8	11 0	.491	8446 0	8 6	.564	7880 6	6 9	.637	7415 1	5 9	.710	7023 6	5 0	.783	6688 3	4 3
.419	9142 9	10 9	.492	8437 4	8 6	.565	7873 6	7 0	.638	7409 3	5 8	.711	7018 6	5 0	.784	6684 0	4 3
.420	9132 0	10 9	.493	8428 9	8 5	.566	7866 6	7 0	.639	7403 5	5 8	.712	7013 7	4 9	.785	6679 7	4 3
.421	9121 2	10 8	.494	8420 4	8 5	.567	7859 7	6 9	.640	7397 8	5 7	.713	7008 8	4 9	.786	6675 5	4 2
.422	9110 3	10 9	.495	8411 9	8 5	.568	7852 8	6 9	.641	7392 0	5 8	.714	7003 9	4 9	.787	6671 2	4 3
.423	9099 6	10 7	.496	8403 4	8 5	.569	7845 9	6 9	.642	7386 3	5 7	.715	6998 9	5 0	.788	6667 0	4 2
0.424	9088 8	10 8	0.497	8394 9	8 5	0 570	7839 0	6 9	0.643	7380 5	5 8	0.716	6994 1	4 8	0.789	6662 8	4 2

TABLE I.—(CONTINUED.)

r^2	$\frac{2\mu}{r}$	Dif.	r^2	$\frac{2\mu}{r}$	Dif.	r^2	$\frac{2\mu}{r}$	Dif.	r^2	$\frac{2\mu}{r}$	Dif.	r^2	$\frac{2\mu}{r}$	Dif.	r^2	$\frac{2\mu}{r}$	Dif.
	0.000			0.000			0.000			0.000			0.000			0.000	
0.789	6662 8		0.862	6374 3		0.935	6120 5		1.08	5694 8		1.81	4399 0		2.54	3713 4	
.790	6658 6	4 2	.863	6370 6	3 7	.936	6117 2	3 3	.09	5668 6	26 2	.82	4386 9	12 1	.55	3706 1	7 3
.791	6654 4	4 2	.864	6367 0	3 6	.937	6114 0	3 2	.10	5642 9	25 7	.83	4374 9	12 0	.56	3698 9	7 2
.792	6650 2	4 2	.865	6363 3	3 7	.938	6110 7	3 3	.11	5617 4	25 5	.84	4363 0	11 9	.57	3691 7	7 2
.793	6646 0	4 2	.866	6359 6	3 7	.939	6107 5	3 2	.12	5592 3	25 1	.85	4351 2	11 8	.58	3684 5	7 2
.794	6641 9	4 1	.867	6356 0	3 6	.940	6104 2	3 3	.13	5567 6	24 7	.86	4339 4	11 8	.59	3677 4	7 1
.795	6637 7	4 2	.868	6352 3	3 7	.941	6101 0	3 2	.14	5543 1	24 5	.87	4327 8	11 6	.60	3670 3	7 1
.796	6633 5	4 2	.869	6348 6	3 7	.942	6097 7	3 3	.15	5518 9	24 2	.88	4316 3	11 5	.61	3663 3	7 0
.797	6629 4	4 1	.870	6345 0	3 6	.943	6094 5	3 2	.16	5495 1	23 8	.89	4304 8	11 5	.62	3656 3	7 0
.798	6625 2	4 2	.871	6341 4	3 6	.944	6091 3	3 2	.17	5471 5	23 6	.90	4293 5	11 3	.63	3649 3	7 0
.799	6621 1	4 1	.872	6337 7	3 7	.945	6088 1	3 2	.18	5448 3	23 2	.91	4282 3	11 2	.64	3642 4	6 9
.800	6616 9	4 2	.873	6334 1	3 6	.946	6084 9	3 2	.19	5425 3	23 0	.92	4271 1	11 2	.65	3635 5	6 9
.801	6612 8	4 1	.874	6330 5	3 6	.947	6081 6	3 3	.20	5402 6	22 7	.93	4260 0	11 1	.66	3628 7	6 8
.802	6608 7	4 1	.875	6326 9	3 6	.948	6078 4	3 2	.21	5380 3	22 3	.94	4249 0	11 0	.67	3621 9	6 8
.803	6604 5	4 2	.876	6323 3	3 6	.949	6075 2	3 2	.22	5358 1	22 2	.95	4238 1	10 9	.68	3615 1	6 8
.804	6000 4	4 1	.877	6319 7	3 6	.950	6072 0	3 2	.23	5336 3	21 8	.96	4227 3	10 8	.69	3608 4	6 7
.805	6596 4	4 0	.878	6316 1	3 6	.951	6068 8	3 2	.24	5314 7	21 6	.97	4216 5	10 8	.70	3601 7	6 7
.806	6592 3	4 1	.879	6312 5	3 6	.952	6065 6	3 2	.25	5293 4	21 3	.98	4205 9	10 6	.71	3595 0	6 7
.807	6588 2	4 1	.880	6308 9	3 6	.953	6062 4	3 2	.26	5272 4	21 0	.99	4195 3	10 6	.72	3588 4	6 6
.808	6584 1	4 1	.881	6305 3	3 6	.954	6059 3	3 1	.27	5251 6	20 8	2.00	4184 8	10 5	.73	3581 9	6 5
.809	6580 0	4 1	.882	6301 6	3 7	.955	6056 1	3 2	.28	5231 0	20 6	.01	4174 4	10 4	.74	3575 3	6 6
.810	6575 9	4 1	.883	6298 2	3 4	.956	6052 9	3 2	.29	5210 7	20 3	.02	4164 1	10 3	.75	3568 8	6 5
.811	6571 8	4 1	.884	6294 7	3 5	.957	6049 8	3 1	.30	5190 6	20 1	.03	4153 8	10 3	.76	3562 4	6 4
.812	6567 8	4 0	.885	6291 1	3 6	.958	6046 6	3 2	.31	5170 7	19 9	.04	4143 6	10 2	.77	3556 0	6 4
.813	6563 8	4 0	.886	6287 5	3 6	.959	6043 5	3 1	.32	5151 1	19 6	.05	4133 5	10 1	.78	3549 5	6 5
.814	6559 7	4 1	.887	6283 9	3 6	.960	6040 3	3 2	.33	5131 7	19 4	.06	4123 5	10 0	.79	3543 2	6 3
.815	6555 7	4 0	.888	6280 4	3 5	.961	6037 2	3 1	.34	5112 5	19 2	.07	4113 5	10 0	.80	3536 8	6 4
.816	6551 7	4 0	.889	6276 8	3 6	.962	6034 0	3 2	.35	5093 5	19 0	.08	4103 6	9 9	.81	3530 5	6 3
.817	6547 7	4 0	.890	6273 3	3 5	.963	6030 9	3 1	.36	5074 8	18 7	.09	4093 8	9 8	.82	3524 2	6 3
.818	6543 7	4 0	.891	6269 8	3 5	.964	6027 8	3 1	.37	5056 2	18 6	.10	4084 0	9 8	.83	3518 0	6 2
.819	6539 7	4 0	.892	6266 3	3 5	.965	6024 6	3 2	.38	5037 9	18 3	.11	4074 3	9 7	.84	3511 8	6 2
.820	6535 6	4 1	.893	6262 7	3 6	.966	6021 5	3 1	.39	5019 7	18 2	.12	4064 7	9 6	.85	3505 6	6 2
.821	6531 6	4 0	.894	6259 2	3 5	.967	6018 4	3 1	.40	5001 8	17 9	.13	4055 2	9 5	.86	3499 5	6 1
.822	6527 6	4 0	.895	6255 7	3 5	.968	6015 3	3 1	.41	4984 0	17 8	.14	4045 7	9 5	.87	3493 4	6 1
.823	6523 7	3 9	.896	6252 2	3 5	.969	6012 2	3 1	.42	4966 4	17 6	.15	4036 3	9 4	.88	3487 3	6 1
.824	6519 7	4 0	.897	6248 7	3 5	.970	6009 1	3 1	.43	4949 0	17 4	.16	4026 9	9 4	.89	3481 3	6 0
.825	6515 7	4 0	.898	6245 3	3 4	.971	6006 0	3 1	.44	4931 8	17 2	.17	4017 6	9 3	.90	3475 3	6 0
.826	6511 8	3 9	.899	6241 8	3 5	.972	6002 9	3 1	.45	4914 8	17 0	.18	4008 4	9 2	.91	3469 3	6 0
.827	6507 9	3 9	.900	6238 3	3 5	.973	5999 8	3 1	.46	4897 9	16 9	.19	3999 3	9 1	.92	3463 4	5 9
.828	6503 9	4 0	.901	6234 8	3 5	.974	5996 7	3 1	.47	4881 2	16 7	.20	3990 1	9 2	.93	3457 5	5 9
.829	6500 0	3 9	.902	6231 4	3 4	.975	5993 6	3 1	.48	4864 7	16 5	.21	3981 1	9 0	.94	3451 6	5 9
.830	6496 0	4 0	.903	6227 9	3 5	.976	5990 6	3 0	.49	4848 4	16 3	.22	3972 1	9 0	.95	3445 8	5 8
.831	6492 1	3 9	.904	6224 5	3 4	.977	5987 5	3 1	.50	4832 2	16 2	.23	3963 2	8 9	.96	3439 9	5 9
.832	6488 2	3 9	.905	6221 0	3 5	.978	5984 4	3 1	.51	4816 2	16 0	.24	3954 3	8 9	.97	3434 1	5 8
.833	6484 3	3 9	.906	6217 6	3 4	.979	5981 4	3 0	.52	4800 3	15 9	.25	3945 5	8 8	.98	3428 4	5 7
.834	6480 4	3 9	.907	6214 1	3 5	.980	5978 3	3 1	.53	4784 6	15 7	.26	3936 8	8 7	.99	3422 6	5 8
.835	6476 5	3 9	.908	6210 7	3 4	.981	5975 3	3 0	.54	4769 1	15 5	.27	3928 0	8 8	3.00	3416 9	5 7
.836	6472 6	3 9	.909	6207 3	3 4	.982	5972 2	3 1	.55	4753 7	15 4	.28	3919 4	8 6	.01	3411 2	5 7
.837	6468 8	3 8	.910	6203 9	3 4	.983	5969 2	3 0	.56	4738 4	15 3	.29	3910 9	8 5	.02	3405 6	5 6
.838	6464 9	3 9	.911	6200 5	3 4	.984	5966 1	3 1	.57	4723 3	15 1	.30	3902 4	8 5	.03	3399 9	5 7
.839	6461 1	3 8	.912	6197 1	3 4	.985	5963 1	3 0	.58	4708 3	15 0	.31	3893 9	8 5	.04	3394 3	5 6
.840	6457 2	3 9	.913	6193 7	3 4	.986	5960 1	3 0	.59	4693 5	14 8	.32	3885 5	8 4	.05	3388 8	5 5
.841	6453 4	3 8	.914	6190 3	3 4	.987	5957 0	3 1	.60	4678 8	14 7	.33	3877 0	8 5	.06	3383 2	5 6
.842	6449 5	3 9	.915	6186 9	3 4	.988	5954 0	3 0	.61	4664 3	14 5	.34	3868 9	8 1	.07	3377 7	5 5
.843	6445 7	3 8	.916	6183 6	3 3	.989	5951 0	3 0	.62	4649 8	14 5	.35	3860 6	8 3	.08	3372 2	5 5
.844	6441 9	3 8	.917	6180 2	3 4	.990	5948 0	3 0	.63	4635 5	14 3	.36	3852 4	8 2	.09	3366 7	5 5
.845	6438 1	3 8	.918	6176 8	3 4	.991	5945 0	3 0	.64	4621 4	14 1	.37	3844 3	8 1	.10	3361 3	5 4
.846	6434 3	3 8	.919	6173 5	3 3	.992	5942 0	3 0	.65	4607 4	14 0	.38	3836 3	8 0	.11	3355 9	5 4
.847	6430 5	3 8	.920	6170 1	3 4	.993	5939 0	3 0	.66	4593 5	13 9	.39	3828 2	8 1	.12	3350 5	5 4
.848	6426 7	3 8	.921	6166 8	3 3	.994	5936 0	3 0	.67	4579 7	13 8	.40	3820 2	8 0	.13	3345 1	5 4
.849	6422 9	3 8	.922	6163 4	3 4	.995	5933 0	3 0	.68	4566 0	13 7	.41	3812 3	7 9	.14	3339 8	5 3
.850	6419 1	3 8	.923	6160 1	3 3	.996	5930 1	2 9	.69	4552 5	13 5	.42	3804 4	7 9	.15	3334 5	5 3
.851	6415 3	3 8	.924	6156 8	3 3	.997	5927 1	3 0	.70	4539 1	13 4	.43	3796 6	7 8	.16	3329 2	5 3
.852	6411 6	3 7	.925	6153 4	3 4	.998	5924 1	3 0	.71	4525 8	13 3	.44	3788 7	7 9	.17	3324 0	5 2
.853	6407 8	3 8	.926	6150 1	3 3	.999	5921 2	2 9	.72	4512 6	13 2	.45	3781 0	7 7	.18	3318 8	5 2
.854	6404 1	3 7	.927	6146 8	3 3	1.000	5918 2	3 0	.73	4499 6	13 0	.46	3773 3	7 7	.19	3313 6	5 2
.855	6400 4	3 7	.928	6143 5	3 3	.010	5888 8	29 4	.74	4486 6	13 0	.47	3765 6	7 7	.20	3308 4	5 2
.856	6396 6	3 8	.929	6140 2	3 3	.020	5859 9	28 9	.75	4473 8	12 8	.45	3758 0	7 6	.21	3303 2	5 2
.857	6392 9	3 7	.930	6136 9	3 3	.030	5831 4	28 5	.76	4461 0	12 8	.49	3750 5	7 5	.22	3298 1	5 1
.858	6389 2	3 7	.931	6133 6	3 3	.040	5803 3	28 1	.77	4448 4	12 6	.50	3743 0	7 5	.23	3293 0	5 1
.859	6385 5	3 7	.932	6130 3	3 3	.050	5775 6	27 7	.78	4435 9	12 5	.51	3735 5	7 5	.24	3287 9	5 1
.860	6381 7	3 8	.933	6127 0	3 3	.060	5748 3	27 3	.79	4423 5	12 4	.52	3728 1	7 4	.25	3282 9	5 0
.861	6378 0	3 7	.934	6123 8	3 2	.070	5721 4	26 9	.80	4411 2	12 3	.53	3720 7	7 4	.26	3277 8	5 1
0.862	6374 3	3 7	0.935	6120 5	3 3	1.080	5694 8	26 6	1.81	4399 0	12 2	2.54	3713 4	7 3	3.27	3272 8	5 0

TABLE I. — (Continued.)

r^2	$\frac{2\mu}{r}$	Dif.	r^2	$\frac{2\mu}{r}$	Dif.	r^2	$\frac{2\mu}{r}$	Dif.	r^2	$\frac{2\mu}{r}$	Dif.	r^2	$\frac{2\mu}{r}$	Dif.	r^2	$\frac{2\mu}{r}$	Dif.
	0.000			0.000			0.000			0.000			0.000			0.000	
3.27	3272 8		3.65	3097 8		4.02	2951 7		4.39	2824 6		4.76	2712 7		6.30	2357 9	
.28	3267 8	5 0	.66	3093 5	4 3	.03	2948 1	3 6	.40	2821 4	3 2	.77	2709 9	2 8	.40	2339 4	18 5
.29	3262 8	5 0	.67	3089 3	4 2	.04	2944 4	3 7	.41	2818 2	3 2	.78	2707 0	2 9	.50	2321 3	18 1
.30	3257 9	4 9	.68	3085 1	4 2	.05	2940 8	3 6	.42	2815 0	3 2	.79	2704 2	2 8	.60	2303 7	17 6
.31	3253 0	4 9	.69	3080 9	4 2	.06	2937 2	3 6	.43	2811 8	3 2	.80	2701 3	2 9	.70	2286 5	17 2
.32	3248 1	4 9	.70	3076 8	4 1	.07	2933 5	3 7	.44	2808 7	3 1	.81	2698 5	2 8	.80	2269 6	16 9
.33	3243 2	4 9	.71	3072 6	4 2	.08	2930 0	3 5	.45	2805 5	3 2	.82	2695 7	2 8	6.90	2253 0	16 6
.34	3238 3	4 9	.72	3068 5	4 1	.09	2926 4	3 6	.46	2802 4	3 1	.83	2692 9	2 8	7.00	2236 8	16 2
.35	3233 5	4 8	.73	3064 4	4 1	.10	2922 8	3 6	.47	2799 2	3 2	.84	2690 2	2 7	.10	2221 0	15 8
.36	3228 7	4 8	.74	3060 3	4 1	.11	2919 3	3 5	.48	2796 1	3 1	.85	2687 4	2 8	.20	2205 6	15 4
.37	3223 9	4 8	.75	3056 2	4 1	.12	2915 7	3 6	.49	2792 0	3 1	.86	2684 6	2 8	.30	2190 4	15 2
.38	3219 1	4 8	.76	3052 1	4 1	.13	2912 2	3 5	.50	2789 9	3 1	.87	2681 9	2 7	.40	2175 6	14 8
.39	3214 3	4 8	.77	3048 1	4 0	.14	2908 7	3 5	.51	2786 8	3 1	.88	2679 1	2 8	.50	2161 0	14 6
.40	3209 6	4 7	.78	3044 0	4 1	.15	2905 2	3 5	.52	2783 7	3 1	.89	2676 4	2 7	.60	2146 8	14 2
.41	3204 9	4 7	.79	3040 0	4 0	.16	2901 7	3 5	.53	2780 7	3 0	.90	2673 6	2 8	.70	2132 8	14 0
.42	3200 2	4 7	.80	3036 0	4 0	.17	2898 2	3 5	.54	2777 6	3 1	.91	2670 9	2 7	.80	2119 1	13 7
.43	3195 5	4 7	.81	3032 0	4 0	.18	2894 7	3 5	.55	2774 6	3 0	.92	2668 2	2 7	7.90	2105 6	13 5
.44	3190 9	4 6	.82	3028 0	4 0	.19	2891 3	3 4	.56	2771 5	3 1	.93	2665 5	2 7	8.00	2092 4	13 2
.45	3186 3	4 6	.83	3024 1	3 9	.20	2887 8	3 5	.57	2768 5	3 0	.94	2662 8	2 7	.10	2079 5	12 9
.46	3181 7	4 6	.84	3020 1	4 0	.21	2884 4	3 4	.58	2765 5	3 0	.95	2660 1	2 7	.20	2066 8	12 7
.47	3177 1	4 6	.85	3016 2	3 9	.22	2881 0	3 4	.59	2762 5	3 0	.96	2657 4	2 7	.30	2054 3	12 5
.48	3172 5	4 6	.86	3012 3	3 9	.23	2877 5	3 5	.60	2759 4	3 1	.97	2654 7	2 7	.40	2042 0	12 3
.49	3167 9	4 6	.87	3008 4	3 9	.24	2874 1	3 4	.61	2756 4	3 0	.98	2652 1	2 6	.50	2029 9	12 1
.50	3163 4	4 5	.88	3004 5	3 9	.25	2870 8	3 3	.62	2753 4	3 0	4.99	2649 4	2 7	.60	2018 1	11 8
.51	3158 9	4 5	.89	3000 7	3 8	.26	2867 4	3 4	.63	2750 5	2 9	5.00	2646 7	2 7	.70	2006 4	11 7
.52	3154 4	4 5	.90	2996 8	3 9	.27	2864 0	3 4	.64	2747 5	3 0	.10	2620 6	26 1	.80	1995 0	11 4
.53	3149 9	4 5	.91	2993 0	3 8	.28	2860 7	3 3	.65	2744 6	2 9	.20	2595 3	25 3	8.90	1983 8	11 2
.54	3145 5	4 4	.92	2989 1	3 9	.29	2857 3	3 4	.66	2741 6	3 0	.30	2570 7	24 6	9.00	1972 7	11 1
.55	3141 1	4 4	.93	2985 4	3 7	.30	2854 0	3 3	.67	2738 7	2 9	.40	2546 8	23 9	.10	1961 9	10 8
.56	3136 6	4 5	.94	2981 6	3 8	.31	2850 7	3 3	.68	2735 8	2 9	.50	2523 6	23 2	.20	1951 2	10 7
.57	3132 2	4 4	.95	2977 8	3 8	.32	2847 4	3 3	.69	2732 9	2 9	.60	2501 0	22 6	.30	1940 6	10 6
.58	3127 9	4 3	.96	2974 0	3 8	.33	2844 1	3 3	.70	2729 9	3 0	.70	2478 9	22 1	.40	1930 3	10 3
.59	3123 5	4 4	.97	2970 3	3 7	.34	2840 8	3 3	.71	2727 0	2 9	.80	2457 4	21 5	.50	1920 1	10 2
.60	3119 2	4 3	.98	2966 6	3 7	.35	2837 6	3 2	.72	2724 1	2 9	5.90	2436 5	20 9	.60	1910 1	10 0
.61	3114 9	4 3	3.99	2962 9	3 7	.36	2834 3	3 3	.73	2721 3	2 8	6.00	2416 1	20 4	.70	1900 3	9 8
.62	3110 6	4 3	4.00	2959 1	3 8	.37	2831 1	3 2	.74	2718 4	2 9	.10	2396 2	19 9	.80	1890 5	9 8
.63	3106 3	4 3	.01	2955 4	3 7	.38	2827 8	3 3	.75	2715 5	2 9	.20	2376 8	19 4	9.90	1880 9	9 6
.64	3102 0	4 3	4.02	2951 7	3 7	4.39	2824 6	3 2	4.76	2712 7	2 8	6.30	2357 9	18 9	10.00	1871 5	9 4
3.65	3097 8	4 2															

TABLE II.

r^2	log. $\frac{\mu}{r^3}$	Dif.	r^2	log. $\frac{\mu}{r^3}$	Dif.	r^2	log. $\frac{\mu}{r^3}$	Dif.	r^2	log. $\frac{\mu}{r^3}$	Dif.	r^2	log. $\frac{\mu}{r^3}$	Dif.	r^2	log. $\frac{\mu}{r^3}$	Dif.
0.351	7.15320		0.375	7.11012		0.399	7.06970		0.423	7.03165		0.447	7.99571		0.471	6.96163	
.352	.15135	185	.376	.10838	174	.400	.06807	163	.424	.03011	154	.448	.99425	146	.472	.96025	138
.353	.14950	185	.377	.10665	173	.401	.06644	163	.425	.02858	153	.449	.99280	145	.473	.95887	138
.354	.14766	184	.378	.10493	172	.402	.06482	162	.426	.02704	154	.450	.99135	145	.474	.95749	138
.355	.14582	184	.379	.10321	172	.403	.06320	162	.427	.02552	152	.451	.98990	145	.475	.95612	137
.356	.14399	183	.380	.10149	172	.404	.06159	161	.428	.02399	153	.452	.98846	144	.476	.95475	137
.357	.14216	183	.381	.09978	171	.405	.05998	161	.429	.02247	152	.453	.98702	144	.477	.95338	137
.358	.14034	182	.382	.09807	171	.406	.05837	161	.430	.02096	151	.454	.98558	144	.478	.95202	136
.359	.13852	182	.383	.09637	170	.407	.05677	160	.431	.01945	151	.455	.98415	143	.479	.95066	136
.360	.13671	181	.384	.09467	170	.408	.05517	160	.432	.01794	151	.456	.98271	144	.480	.94930	136
.361	.13490	181	.385	.09297	170	.409	.05358	159	.433	.01643	151	.457	.98128	143	.481	.94795	135
.362	.13310	180	.386	.09128	169	.410	.05199	159	.434	.01493	150	.458	.97986	142	.482	.94659	136
.363	.13130	180	.387	.08960	168	.411	.05040	159	.435	.01343	150	.459	.97844	142	.483	.94524	135
.364	.12951	179	.388	.08792	168	.412	.04882	158	.436	.01193	150	.460	.97702	142	.484	.94390	134
.365	.12772	179	.389	.08624	168	.413	.04724	158	.437	.01044	149	.461	.97560	142	.485	.94255	135
.366	.12594	178	.390	.08457	167	.414	.04567	157	.438	.00895	149	.462	.97419	141	.486	.94121	134
.367	.12417	177	.391	.08290	167	.415	.04409	158	.439	.00747	148	.463	.97278	141	.487	.93988	133
.368	.12239	178	.392	.08124	166	.416	.04253	156	.440	.00599	148	.464	.97137	141	.488	.93854	134
.369	.12063	176	.393	.07958	166	.417	.04096	157	.441	.00451	148	.465	.96997	140	.489	.93721	133
.370	.11886	177	.394	.07792	166	.418	.03940	156	.442	.00304	147	.466	.96857	140	.490	.93587	134
.371	.11710	176	.395	.07627	165	.419	.03784	156	.443	.00156	148	.467	.96717	140	.491	.93454	133
.372	.11535	175	.396	.07462	165	.420	.03629	155	.444	.00010	146	.468	.96578	139	.492	.93322	132
.373	.11360	175	.397	.07298	164	.421	.03474	155	.445	.99863	147	.469	.96439	139	.493	.93190	132
.374	.11186	174	.398	.07134	164	.422	.03319	155	.446	.99717	146	.470	7.96301	138	.494	.93058	132
0.375	7.11012	174	0.399	7.06970	164	0.423	7.03165	154	0.447	7.99571	146	0.471	6.96163	138	0.495	6.92926	131

TABLE II.—(CONTINUED.)

r^2	log. $\frac{\mu}{r^3}$	Dif.	r^2	log. $\frac{\mu}{r^3}$	Dif.	r^2	log. $\frac{\mu}{r^3}$	Dif.	r^2	log. $\frac{\mu}{r^3}$	Dif.	r^2	log. $\frac{\mu}{r^3}$	Dif.	r^2	log. $\frac{\mu}{r^3}$	Dif.
0 495	6.92926		0.568	6.83964		0.641	6.76087		0.714	6.69061		0.787	6.62720		0.860	6.56941	
.496	.92795	131	.569	.83849	115	.642	.75985	102	.715	.68970	91	.788	.62637	83	.861	.56865	76
.497	.92664	131	.570	.83735	114	.643	.75884	101	.716	.68879	91	.789	.62554	83	.862	.56790	75
.498	.92533	131	.571	.83621	114	.644	.75783	101	.717	.68788	91	.790	.62472	82	.863	.56714	76
.499	.92402	131	.572	.83507	114	.645	.75682	101	.718	.68697	91	.791	.62390	82	.864	.56639	75
.500	.92271	131	.573	.83393	114	.646	.75581	101	.719	.68606	91	.792	.62307	83	.865	.56564	75
.501	.92141	130	.574	.83279	114	.647	.75480	101	.720	.68516	90	.793	.62225	82	.866	.56489	75
.502	.92011	130	.575	.83166	113	.648	.75380	100	.721	.68426	90	.794	.62143	82	.867	.56414	75
.503	.91882	129	.576	.83053	113	.649	.75280	100	.722	.68335	91	.795	.62061	82	.868	.56339	75
.504	.91752	130	.577	.82940	113	.650	.75180	99	.723	.68245	90	.796	.61979	82	.869	.56264	75
.505	.91623	129	.578	.82827	113	.651	.75079	101	.724	.68155	90	.797	.61897	82	.870	.56188	76
.506	.91495	128	.579	.82714	113	.652	.74979	100	.725	.68065	90	.798	.61816	81	.871	.56113	75
.507	.91366	129	.580	.82602	112	.653	.74880	99	.726	.67976	89	.799	.61734	82	.872	.56039	74
.508	.91238	128	.581	.82490	112	.654	.74780	100	.727	.67886	90	.800	.61653	81	.873	.55964	75
.509	.91109	129	.582	.82378	112	.655	.74680	100	.728	.67796	90	.801	.61572	82	.874	.55890	74
.510	.90981	128	.583	.82266	112	.656	.74581	99	.729	.67707	89	.802	.61490	81	.875	.55815	75
.511	.90853	127	.584	.82154	112	.657	.74482	99	.730	.67618	89	.803	.61409	81	.876	.55741	74
.512	.90726	127	.585	.82043	111	.658	.74383	99	.731	.67529	89	.804	.61328	81	.877	.55667	74
.513	.90599	127	.586	.81932	111	.659	.74284	99	.732	.67440	89	.805	.61247	81	.878	.55593	74
.514	.90472	127	.587	.81821	111	.660	.74185	99	.733	.67351	89	.806	.61166	81	.879	.55519	74
.515	.90346	126	.588	.81710	111	.661	.74086	99	.734	.67262	89	.807	.61085	80	.880	.55444	75
.516	.90219	127	.589	.81599	111	.662	.73987	99	.735	.67173	89	.808	.61005	81	.881	.55370	74
.517	.90093	126	.590	.81489	110	.663	.73889	98	.736	.67085	88	.809	.60924	80	.882	.55296	74
.518	.89967	125	.591	.81379	110	.664	.73791	98	.737	.66996	89	.810	.60844	80	.883	.55223	73
.519	.89842	126	.592	.81269	110	.665	.73693	98	.738	.66908	88	.811	.60764	81	.884	.55149	74
.520	.89716	125	.593	.81159	110	.666	.73595	98	.739	.66820	88	.812	.60683	80	.885	.55075	74
.521	.89591	125	.594	.81049	110	.667	.73497	98	.740	.66732	88	.813	.60603	80	.886	.55002	73
.522	.89466	125	.595	.80939	110	.668	.73399	98	.741	.66644	88	.814	.60523	80	.887	.54928	74
.523	.89341	125	.596	.80830	109	.669	.73302	97	.742	.66556	88	.815	.60443	80	.888	.54855	73
.524	.89216	125	.597	.80721	109	.670	.73205	97	.743	.66468	88	.816	.60363	80	.889	.54782	73
.525	.89091	125	.598	.80612	109	.671	.73108	97	.744	.66380	88	.817	.60283	80	.890	.54708	74
.526	.88967	124	.599	.80503	109	.672	.73011	97	.745	.66293	87	.818	.60204	79	.891	.54635	73
.527	.88843	124	.600	.80394	109	.673	.72914	97	.746	.66205	88	.819	.60124	80	.892	.54562	73
.528	.88720	123	.601	.80285	109	.674	.72817	97	.747	.66118	87	.820	.60045	79	.893	.54489	73
.529	.88597	123	.602	.80177	108	.675	.72720	97	.748	.66031	87	.821	.59966	79	.894	.54417	72
.530	.88474	123	.603	.80069	108	.676	.72624	96	.749	.65943	88	.822	.59886	80	.895	.54344	73
.531	.88351	123	.604	.79961	108	.677	.72528	96	.750	.65857	86	.823	.59807	79	.896	.54271	73
.532	.88229	122	.605	.79853	108	.678	.72431	97	.751	.65770	87	.824	.59728	79	.897	.54199	72
.533	.88106	123	.606	.79746	107	.679	.72335	96	.752	.65683	87	.825	.59649	79	.898	.54126	73
.534	.87984	122	.607	.79638	108	.680	.72240	95	.753	.65597	86	.826	.59570	79	.899	.54054	72
.535	.87863	121	.608	.79531	107	.681	.72144	95	.754	.65511	86	.827	.59492	78	.900	.53980	74
.536	.87741	122	.609	.79424	107	.682	.72049	95	.755	.65424	87	.828	.59413	79	.901	.53908	72
.537	.87620	121	.610	.79317	107	.683	.71953	96	.756	.65338	86	.829	.59334	79	.902	.53835	73
.538	.87499	121	.611	.79210	107	684	.71858	95	.757	.65252	86	.830	.59254	80	.903	.53763	72
.539	.87378	121	.612	.79104	106	.685	.71763	95	.758	.65166	86	.831	.59175	79	.904	.53691	72
.540	.87257	121	.613	.78997	107	.686	.71668	95	.759	.65080	86	.832	.59097	78	.905	.53619	72
.541	.87137	120	.614	.78891	106	.687	.71573	95	.760	.64995	85	.833	.59019	78	.906	.53547	72
.542	.87016	121	.615	.78785	106	.688	.71478	95	.761	.64909	86	.834	.58941	78	.907	.53476	71
.543	.86896	120	.616	.78679	106	.689	.71383	95	.762	.64824	85	.835	.58863	78	.908	.53404	72
.544	.86777	119	.617	.78574	105	.690	.71289	94	.763	.64738	86	.836	.58785	78	.909	.53332	72
.545	.86657	120	.618	.78468	106	.691	.71195	94	.764	.64653	85	.837	.58707	78	.910	.53260	72
.546	.86538	119	.619	.78363	105	.692	.71100	95	.765	.64567	86	.838	.58629	78	.911	.53188	72
.547	.86419	119	.620	.78258	105	.693	.71006	94	.766	.64482	85	.839	.58552	77	.912	.53117	71
.548	.86300	119	.621	.78153	105	.694	.70912	94	.767	.64397	85	.840	.58474	78	.913	.53046	71
.549	.86181	119	.622	.78048	105	.695	.70818	94	.768	.64312	85	.841	.58396	78	.914	.52975	71
.550	.86062	118	.623	.77943	105	.696	.70724	94	.769	.64227	85	.842	.58319	78	.915	.52903	72
.551	.85944	118	.624	.77839	104	.697	.70631	93	.770	.64143	84	.843	.58242	77	.916	.52832	71
.552	.85826	118	.625	.77735	104	.698	.70537	94	.771	.64058	85	.844	.58165	77	.917	.52761	71
.553	.85708	118	.626	.77630	105	.699	.70444	93	.772	.63974	84	.845	.58088	77	.918	.52690	71
.554	.85590	118	.627	.77526	104	.700	.70351	93	.773	.63889	85	.846	.58010	78	.919	.52619	71
.555	.85472	118	.628	.77422	104	.701	.70258	93	.774	.63805	84	.847	.57933	77	.920	.52548	71
.556	.85355	117	.629	.77319	103	.702	.70165	93	.775	.63721	84	.848	.57856	77	.921	.52477	71
.557	.85238	117	.630	.77215	104	.703	.70072	93	.776	.63637	84	.849	.57779	77	.922	.52406	71
.558	.85121	117	.631	.77112	103	.704	.69980	92	.777	.63553	84	.850	.57703	76	.923	.52336	70
.559	.85004	117	.632	.77008	104	.705	.69887	93	.778	.63469	84	.851	.57626	77	.924	.52265	71
.560	.84888	116	.633	.76905	103	.706	.69795	92	.779	.63385	84	.852	.57550	76	.925	.52195	70
.561	.84772	116	.634	.76802	103	.707	.69703	92	.780	.63302	83	.853	.57474	76	.926	.52125	70
.562	.84656	116	.635	.76700	102	.708	.69611	92	.781	.63218	84	.854	.57397	77	.927	.52054	71
.563	.84540	116	.636	.76597	103	.709	.69519	92	.782	.63135	83	.855	.57321	76	.928	.51984	70
.564	.84424	116	.637	.76495	102	.710	.69427	92	.783	.63052	83	.856	.57245	76	.929	.51914	70
.565	.84309	115	.638	.76393	102	.711	.69335	92	.784	62968	84	.857	.57168	77	.930	.51844	70
.566	.84193	116	.639	.76291	102	.712	.69244	91	.785	.62885	83	.858	.57092	76	.931	.51774	70
.567	.84078	115	.640	.76189	102	.713	.69152	92	.786	.62802	83	.859	.57017	75	.932	.51704	70
0.568	6.83964	114	0.641	6.76087	102	0.714	6.69061	91	0.787	6.62720	82	0.860	6 56941	76	0.933	6.51634	70

TABLE II.—(CONTINUED.)

r^2	log. $\frac{\mu}{r^3}$	Dif.	r^2	log. $\frac{\mu}{r^3}$	Dif.	r^2	log. $\frac{\mu}{r^3}$	Dif.	r^2	log. $\frac{\mu}{r^3}$	Dif.	r^2	log. $\frac{\mu}{r^3}$	Dif.	r^2	log. $\frac{\mu}{r^3}$	Dif.
0.933	6.51634		1.06	6.43320		1.79	6.09189		2.52	5.86906		3.25	5.70334		3.98	5.57134	
.934	.51564	70	.07	.42709	611	.80	.08826	363	.53	.86648	258	.26	.70134	200	3.99	.56970	164
.935	.51495	69	.08	.42103	606	.81	.08465	361	.54	.86391	257	.27	.69934	200	4 00	.56807	163
.936	.51425	70	.09	.41503	600	.82	.08106	359	.55	.86135	256	.28	.69736	208	.01	.56644	163
.937	.51356	69	.10	.40908	595	.83	.07749	357	.56	.85880	255	.29	.69537	209	.02	.56482	162
.938	.51286	70	.11	.40318	590	.84	.07394	355	.57	.85626	254	.30	.69340	197	.03	.56320	162
.939	.51217	69	.12	.39733	585	.85	.07041	353	.58	.85373	253	.31	.69143	197	.04	.56159	161
.940	.51147	70	.13	.39154	579	.86	.06690	351	.59	.85121	252	.32	.68946	197	.05	.55998	161
.941	.51078	69	.14	.38580	574	.87	.06340	350	.60	.84871	250	.33	.68750	196	.06	.55837	161
.942	.51009	69	.15	.38011	569	.88	.05993	347	.61	.84621	250	.34	.68555	195	.07	.55677	160
.943	.50940	69	.16	.37447	564	.89	.05647	346	.62	.84372	249	.35	.68360	195	.08	.55517	160
.944	.50871	69	.17	.36888	559	.90	.05304	343	.63	.84123	249	.36	.68166	194	.09	.55358	159
.945	.50802	69	.18	.36334	554	.91	.04962	342	.64	.83876	247	.37	.67972	194	.10	.55199	159
.946	.50733	69	.19	.35784	550	.92	.04622	340	.65	.83630	246	.38	.67779	193	.11	.55040	159
.947	.50664	69	.20	.35239	555	.93	.04283	339	.66	.83384	246	.39	.67586	193	.12	.54882	158
.948	.50595	69	.21	.34698	541	.94	.03947	336	.67	.83140	244	.40	.67394	192	.13	.54724	158
.949	.50527	68	.22	.34162	536	.95	.03612	335	.68	.82896	244	.41	.67203	191	.14	.54567	157
.950	.50458	69	.23	.33630	532	.96	.03278	334	.69	.82654	242	.42	.67012	191	.15	.54409	158
.951	.50389	69	.24	.33103	527	.97	.02947	331	.70	.82412	242	.43	.66821	191	.16	.54253	156
.952	.50321	68	.25	.32580	523	.98	.02617	330	.71	.82171	241	.44	.66632	189	.17	.54076	157
.953	.50253	68	.26	.32061	519	1.99	.02288	329	.72	.81931	240	.45	.66442	190	.18	.53940	156
.954	.50185	68	.27	.31546	515	2.00	.01962	326	.73	.81692	239	.46	.66254	188	.19	.53784	156
.955	.50116	69	.28	.31035	511	.01	.01637	325	.74	.81454	238	.47	.66066	188	.20	.53629	155
.956	.50048	68	.29	.30528	507	.02	.01314	323	.75	.81216	238	.48	.65879	187	.21	.53474	155
.957	.49980	68	.30	.30025	503	.03	.00992	322	.76	.80980	236	.49	.65692	187	.22	.53319	155
.958	.49912	68	.31	.29526	499	.04	.00671	321	.77	.80744	236	.50	.65506	186	.23	.53165	154
.959	.49844	68	.32	.29031	495	.05	.00353	318	.78	.80509	235	.51	.65320	186	.24	.53011	154
.960	.49776	68	.33	.28539	492	.06	6.00035	318	.79	.80275	234	.52	.65135	185	.25	.52858	153
.961	.49708	68	.34	.28051	488	.07	5.99720	315	.80	.80042	233	.53	.64950	185	.26	.52704	154
.962	.49641	67	.35	.27566	485	.08	.99406	314	.81	.79810	232	.54	.64766	184	.27	.52552	152
.963	.49573	68	.36	.27086	480	.09	.99093	313	.82	.79578	232	.55	.64582	184	.28	.52399	153
.964	.49505	68	.37	.26608	478	.10	.98783	310	.83	.79348	230	.56	.64399	183	.29	.52247	152
.965	.49438	67	.38	.26135	473	.11	.98473	310	.84	.79118	230	.57	.64216	183	.30	.52096	151
.966	.49371	67	.39	.25664	471	.12	.98166	307	.85	.78889	229	.58	.64034	182	.31	.51945	151
.967	.49303	68	.40	.25197	467	.13	.97859	307	.86	.78661	228	.59	.63852	182	.32	51794	151
.968	.49236	67	.41	.24734	463	.14	.97554	305	.87	.78433	228	.60	.63671	181	.33	.51643	151
.969	.49169	67	.42	.24273	461	.15	.97250	304	.88	.78207	226	.61	.63490	181	.34	.51493	150
.970	.49101	68	.43	.23816	457	.16	.96948	302	.89	.77981	226	.62	.63310	180	.35	.51343	150
.971	.49034	67	.44	.23363	453	.17	.96647	301	.90	.77756	225	.63	.63130	180	.36	.51193	150
.972	.48967	67	.45	.22912	451	.18	.96347	300	.91	.77532	224	.64	.62951	179	.37	.51044	149
.973	.48900	67	.46	.22464	448	.19	.96049	308	.92	.77308	224	.65	.62772	179	.38	.50895	149
.974	.48832	68	.47	.22020	444	.20	.95753	296	.93	.77086	222	.66	.62594	178	.39	.50747	148
.975	.48765	67	.48	.21578	442	.21	.95458	295	.94	.76864	222	.67	.62417	177	.40	.50599	148
.976	.48698	67	.49	.21139	439	.22	.95164	294	.95	.76642	222	.68	.62239	178	.41	.50451	148
.977	.48632	66	.50	.20703	436	.23	.94871	293	.96	.76422	220	.69	.62063	176	.42	.50304	147
.978	.48565	67	.51	.20270	433	.24	.94579	292	.97	.76202	220	.70	.61886	177	.43	.50156	148
.979	.48498	67	.52	.19840	430	.25	.94289	290	.98	.75983	219	.71	.61710	176	.44	.50010	146
.980	.48432	66	.53	.19413	427	.26	.94001	288	2.99	.75765	218	.72	.61535	175	.45	.49863	147
.981	.48365	67	.54	.18989	424	.27	.93713	288	3.00	.75548	217	.73	.61360	175	.46	.49717	146
.982	.48299	66	.55	.18567	422	.28	.93427	286	.01	.75331	217	.74	.61186	174	.47	.49571	146
.983	.48233	66	.56	.18148	419	.29	.93142	285	.02	.75115	216	.75	.61012	174	.48	.49425	146
.984	.48166	67	.57	.17732	416	.30	.92858	284	.03	.74900	215	.76	.60838	174	.49	.49280	145
.985	.48100	66	.58	.17318	414	.31	.92575	283	.04	.74685	215	.77	.60665	173	.50	.49135	145
.986	.48034	66	.59	.16907	411	.32	.92294	281	.05	.74472	213	.78	.60493	172	.51	.48990	145
.987	.47968	66	.60	.16499	408	.33	.92014	280	.06	.74258	214	.79	.60321	172	.52	.48846	144
.988	.47902	66	.61	.16093	406	.34	.91735	279	.07	.74046	212	.80	.60149	172	.53	.48702	144
.989	.47836	66	.62	.15690	403	.35	.91457	278	.08	.73834	212	.81	.59978	171	.54	.48558	144
.990	.47770	66	.63	.15289	401	.36	.91180	277	.09	73623	211	82	.59807	171	.55	.48415	143
.991	.47704	66	.64	.14890	399	.37	.90905	275	.10	.73412	211	.83	.59637	170	.56	.48271	144
.992	.47638	66	.65	.14494	396	.38	.90630	275	.11	.73202	210	.84	.59467	170	.57	.48128	143
.993	.47573	65	.66	.14100	394	.39	.90357	273	12	.72993	209	.85	.59297	170	.58	.47986	142
.994	.47507	66	.67	.13709	391	.40	.90085	272	.13	.72785	208	.86	.59128	169	.59	.47844	142
.995	.47442	65	.68	.13320	389	.41	.89814	271	.14	.72577	208	.87	.58960	168	.60	.47702	142
.996	.47377	65	.69	.12933	387	.42	.89544	270	.15	.72670	207	.88	.58792	168	.61	.47560	142
.997	.47311	66	.70	.12549	384	.43	.89276	268	.16	.72163	207	.89	.58624	168	.62	.47419	141
.998	.47246	65	.71	.12167	399	.44	.89008	268	.17	.71957	206	.90	.58457	167	.63	.47278	141
0.999	.47181	65	.72	.11787	380	.45	.88742	266	.18	.71752	2 5	.91	.58290	167	.64	.47137	141
1.000	.47116	65	.73	.11410	377	46	.88476	266	.19	.71548	204	.92	.58124	166	.65	.46997	140
.010	.46468	648	.74	.11034	376	.47	.88212	264	.20	.71344	204	.93	57958	166	.66	.46857	140
.020	.45826	642	.75	.10661	373	.48	.87949	263	.21	.71141	203	.94	.57792	166	.67	.46717	140
.030	.45190	636	.76	.10290	371	.49	.87687	262	.22	.70938	203	.95	.57627	165	.68	.46578	139
.040	.44561	629	.77	.09921	369	.50	.87425	262	.23	.70736	202	.96	.57462	165	.69	.46439	139
.050	.43938	623	.78	.09554	367	.51	87165	260	.24	.70535	201	97	57298	164	.70	46301	138
1.060	6.43320	618	1.79	6.09189	365	2.52	5.86906	259	3.25	5.70334	201	3 98	5.57134	164	4.71	5.46163	138

TABLE II. — (CONTINUED.)

r^2	log. $\frac{\mu}{r^3}$	Dif.	r^2	log. $\frac{\mu}{r^3}$	Dif.	r^2	log. $\frac{\mu}{r^3}$	Dif.	r^2	log. $\frac{\mu}{r^3}$	Dif.	r^2	log. $\frac{\mu}{r^3}$	Dif.	r^2	log. $\frac{\mu}{r^3}$	Dif.
4.71	5.46163		5.04	5.41752		5.37	5.37620		5.70	5.33735		6.03	5.30069		6.36	5.26597	
.72	.46025	138	.05	.41623	129	.38	.37499	121	.71	.33621	114	.04	.29961	108	.37	.26495	102
.73	.45887	138	.06	.41495	128	.39	.37378	121	.72	.33507	114	.05	.29854	107	.38	.26393	102
.74	.45749	138	.07	.41366	129	.40	.37257	121	.73	.33394	113	.06	.29746	108	.39	.26291	102
.75	.45612	137	.08	.41238	128	.41	.37137	120	.74	.33280	114	.07	.29639	107	.40	.26189	102
.76	.45475	137	.09	.41109	129	.42	.37016	119	.75	.33166	114	.08	.29531	108	.41	.26088	101
.77	.45338	137	.10	.40981	128	.43	.36896	120	.76	.33053	113	.09	.29424	107	.42	.25986	102
.78	.45202	136	.11	.40853	128	.44	.36777	119	.77	.32940	113	.10	.29317	107	.43	.25885	101
.79	.45066	136	.12	.40726	127	.45	.36657	120	.78	.32827	113	.11	.29210	107	.44	.25784	101
.80	.44930	136	.13	.40599	127	.46	.36538	119	.79	.32714	113	.12	.29103	107	.45	.25683	101
.81	.44795	135	.14	.40472	127	.47	.36419	119	.80	.32602	112	.13	.28997	106	.46	.25582	101
.82	.44659	136	.15	.40346	126	.48	.36300	119	.81	.32490	112	.14	.28891	106	.47	.25481	101
.83	.44524	135	.16	.40219	127	.49	.36181	119	.82	.32378	112	.15	.28785	106	.48	.25381	100
.84	.44390	134	.17	.40093	126	.50	.36062	119	.83	.32266	112	.16	.28680	105	.49	.25280	101
.85	.44255	135	.18	.39967	126	.51	.35944	118	.84	.32155	111	.17	.28574	106	.50	.25180	100
.86	.44121	134	.19	.39842	125	.52	.35825	119	.85	.32043	112	.18	.28469	105	.51	.25080	100
.87	.43988	133	.20	.39716	126	.53	.35707	118	.86	.31932	111	.19	.28363	106	.52	.24980	100
.88	.43854	134	.21	.39591	125	.54	.35589	118	.87	.31821	111	.20	.28258	105	.53	.24880	100
.89	.43721	133	.22	.39466	125	.55	.35472	117	.88	.31710	111	.21	.28153	105	.54	.24781	99
.90	.43587	134	.23	.39341	125	56	.35354	118	.89	.31600	110	.22	.28048	105	.55	.24681	100
.91	.43454	133	.24	.39216	135	.57	.35237	117	.90	.31489	111	.23	.27944	104	.56	.24582	99
.92	.43322	132	.25	.39091	125	.58	.35121	116	.91	.31379	110	.24	.27840	104	.57	·24482	100
.93	.43190	132	.26	.38967	124	.59	.35004	117	.92	.31269	110	25	.27735	105	.58	.24383	99
.94	.43058	132	.27	.38843	124	.60	.34888	116	.93	.31159	110	.26	.27631	104	.59	.24284	99
.95	.42926	132	.28	.38720	123	.61	.34772	116	.94	.31049	110	.27	.27527	104	.60	.24185	99
.96	.42795	131	.29	.38597	123	.62	.34656	116	.95	.30939	110	.28	.27423	104	.61	.24086	99
.97	.42664	131	.30	.38474	123	.63	.34541	115	.96	.30830	109	.29	.27319	104	.62	.23988	98
.98	.42533	131	.31	.38351	123	.64	.34425	116	.97	.30721	109	.30	.27215	104	.63	.23890	98
.99	.42402	131	.32	.38229	122	.65	.34309	116	.98	.30612	109	.31	.27112	103	.64	.23792	98
5.00	.42271	131	.33	.38106	123	.66	.34194	115	5.99	.30503	109	.32	.27008	104	.65	.23693	99
01	.42141	130	.34	.37984	122	.67	.34079	115	6.00	.30394	109	.33	.26905	103	.66	.23595	98
.02	.42011	130	.35	.37863	121	.68	.33964	115	.01	.30285	109	.34	.26802	103	.67	.23497	98
.03	.41882	129	.36	.37741	122	.69	.33849	115	.02	.30177	108	.35	.26700	102	.68	.23399	98
5.04	5.41752	130	5.37	5.37 20	121	5.70	5.33735	114	6.03	5.30069	108	6.36	5.26597	103	6.69	5.23302	97